Effectiveness and Addictiveness of Quantitative Easing*

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Abstract

This paper analyzes optimal asset-purchase policies in a macroeconomic model with banks, which face occasionally-binding balance-sheet constraints. It proves analytically that asset-purchase policies are effective in offsetting large financial disturbances, which impair banks' capital position. It warns, however, that the policy can remain ineffective after non-financial shocks and might offer no substitute for interest rate policy when the latter is constrained by the lower bound. Furthermore, the asset-purchase policy is addictive because it flattens the yield curve, reduces the profitability of the banking sector, and therefore slows down its recapitalization. Consequently, the optimal exit from large central bank balance sheets is gradual.

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JEL codes: E32, E44, E52

1 Introduction

Many of the world's leading reserve banks, including the US Federal Reserve and the European Central Bank, have built up large balance sheets to mitigate the macroeconomic fallout of recent financial disturbances, such as the default of Lehman Brothers or the European sovereign debt crisis. A notable feature of these balance sheet policies is their persistence: even though financial conditions and the macroeconomic environment have turned benign in the decade following the crisis, the central bank balance sheets remained large and were being reduced only gradually, if at all.

Figure 1 illustrates this point. It shows the evolution of the central bank balance sheets and the parallel evolution of two relevant measures of credit conditions: a high-yield corporate

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bond spread and the excess bond premium in the US and the euro area. The spread shows the evolution of the costs of corporate borrowing relative to government bond yields of the same maturity, and the excess bond premium additionally purges the spread from the impact of regular default compensation. The excess persistence of the balance sheets is apparent, especially in the face of low excess bond premia.

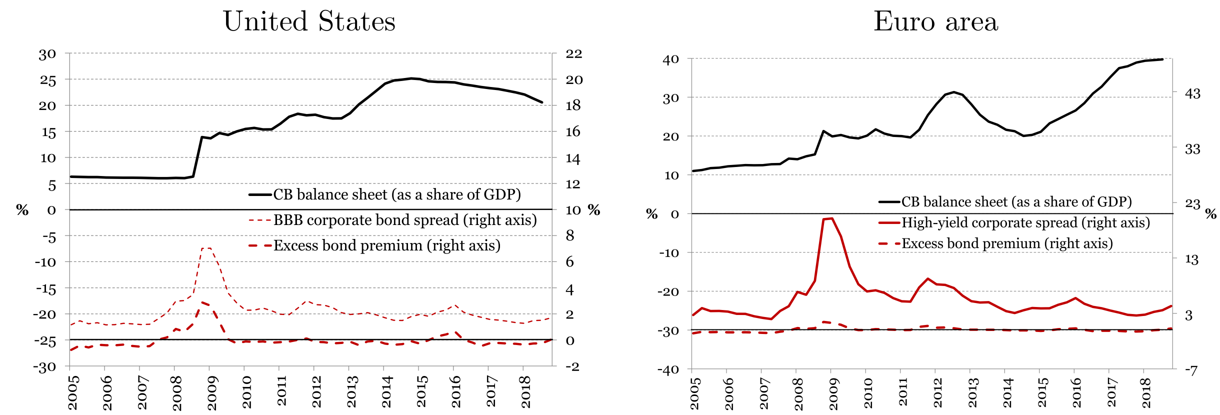

Figure 1: Central bank balance sheets and corporate credit spreads in the US and euro area
Note: The figures plot the evolution of the balance sheet of the US Federal Reserve and the European Central Bank as a fraction of US and euro area annual GDP, the Bank of America - Merrill Lynch high-yield option-adjusted corporate bond spreads, and measures of the excess bond premia in the US (Gilchrist and Zakrajsek, 2012) and in the euro area (De Santis, 2018).

Is such a gradual exit strategy optimal? Should the central bank keep its balance sheet size elevated to avoid returning to an adverse financial environment, or should it reduce the size of its balance sheet quickly to minimize market distortions? In the future, should balance sheet policies become a part of the standard central banking toolkit, and are they necessarily available substitutes when the interest rate policy is constrained by its lower bound? We address these questions through the lens of a macroeconomic model.

In our model, the central bank purchases long-term government bonds (QE) optimally under commitment. QE can be effective, because financial intermediaries (or, simply, banks) face balance sheet constraints (Gertler and Karadi, 2011), which occasionally bind. When the banks' constraints bind, QE eases credit conditions through freeing up banks' balance sheet capacity and allowing banks to extend extra credit to the private sector. When banks' constraints are loose, however, additional QE is ineffective because credit is ample and QE fully crowds out private lending. In our calibration, we assume that the latter benign conditions describe the steady state; so, absent disturbances, the optimal central bank portfolio of long-term government bonds is zero.[1] Adverse financial disturbances, however, can push banks to their constraints and lead to impaired credit conditions and a macroeconomic downturn. These financial disturbances are the focus of our analysis, and we model them as exogenous

[1]This is a conservative assumption that pits our calibration against asset purchase policies. If financial constraints are binding also in steady state, then QE would be used optimally also in the long term.

shocks to banks' equity capital. These shocks can be caused by fluctuations in the valuation in the banks' assets.[2]

We find that the central bank optimally responds to adverse financial disturbances with asset purchase policies *only* and keeps interest rates unchanged, if QE policies are costless. The reason is that QE policies are better suited to mitigating the root cause of the disturbance, which is the credit crunch caused by the tight balance sheet constraints of banks. We characterize the optimal QE analytically. We find that the optimal QE policy fully offsets the macroeconomic impact of the shock: financial and economic conditions stay as benign and efficient as in the steady state. To achieve this, the central bank follows a targeting rule that mirrors the capital shortfall of the banking sector and terminates its asset-purchase program as soon as the capital shortfall disappears. The policy eases banks' balance sheet constraints and allows them to fully satisfy all credit demand while they rebuild their equity capital. In this sense, QE is effective.

At the same time, QE is addictive. The reason is that optimal QE policy slows down bank recapitalization: the benign credit conditions and competition between banks reduce banks' profitability, and, therefore, curtails their means and incentives to rebuild equity capital quickly. The slow recapitalization of the banking sector implies a very gradual exit from the large central bank balance sheet. However, the slow bank recapitalization is not detrimental for welfare, because the central bank can costlessly offset its impact on credit supply. The results imply that the central bank optimally maintains a positive balance sheet long after the banks' balance sheet constraint stops binding. This may sound surprising: when banks' balance sheet constraints are loose, the reduction of the CB's balance sheet does not have any negative impact on the margin. But financial conditions are benign in the first place *precisely* because of the large central bank balance sheet: without it, credit supply would be limited. Therefore, a quick reduction of the central bank's balance sheet would be suboptimal, as it would bring back the impaired credit conditions.

We assess the robustness of our findings in case of positive QE costs. In particular, we introduce a small quadratic efficiency cost to QE as a reduced-form proxy for unmodeled distortions and political costs of maintaining a positive central bank balance sheet. The optimal exit from QE remains very gradual. Curiously, the costs change the optimal path of the phasing-in of QE. The optimal entry is delayed because doing no QE in the presence of deteriorating credit conditions – while it is detrimental to the general welfare and worsens the impact of the shock on banks' balance sheet initially – speeds up the recapitalization of the banking sector by providing banks excess premium on their lending activity. The quicker recapitalization reduces the necessary size of *future* central bank balance sheets. Therefore, the central bank is willing to tolerate short term welfare losses in return for future savings on the cost of QE. The optimal exit remains gradual also if the interest rate policy has a sizable room for maneuver to

[2] Such fluctuations necessarily occur in a financial downturn, that can be brought about by a heightened distrust in banks, which force them to reduce their leverage (Jermann and Quadrini, 2012), or by an unexpected drop in or negative news about the future evolution of capital quality (Gertler and Karadi, 2011).

complement QE policy. We find that the optimal interest rate eases substantially only for a short period to counterbalance the negative impact of the credit crunch on the economy, but QE remains the main credit easing instrument in later periods.

How should the central bank use QE policies in the future? Our non-linear model allows us to assess the welfare gains from QE interventions. We draw two main conclusions.

First, the nature of the shock matters. On the one hand, QE is an effective tool to offset the negative impact of financial shocks even if the interest rate policy is unconstrained by the lower bound. QE is preferable because it directly mitigates the credit crunch. If the costs of QE are small, then QE alone might be sufficient to stabilize the economy and there might be no need for any accompanying interest rate easing. On the other hand, however, QE intervention is not necessarily justified as a response to alternative shocks that do not disrupt the supply of credit, even if the interest rate reaction is restricted. We present an example of a demand shock, which causes a downturn and even drives the interest rate to its lower bound but still *improves* banks' balance sheets. Therefore, credit supply remains abundant and QE stays ineffective. The example serves as a warning: QE is not necessarily available as an effective policy tool to substitute interest rate policy that is constrained by the lower bound.

Second, the size of the shock matters. After small shocks, even of a financial origin, a sizable QE can be welfare detrimental because the efficiency costs exceed any credit easing gains. However, when a shock becomes large and the interest rate reaction becomes sufficiently restricted, then the balance-sheet health of the banking sector necessarily worsens even if the shock is not of a financial origin. In these cases, asset purchase policies again become an important additional tool for monetary policy easing.

Related literature. Our paper is related to an active ongoing research area that assesses optimal asset-purchase policies in dynamic stochastic general equilibrium models. There are two key differences in our framework relative to the literature. First, we analyze an environment, in which banks' balance sheet constraints bind only *occasionally*, so asset purchases are not always effective. In contrast, most of the related literature assumes that QE is always effective, and can reliably substitute interest rate policy, which is constrained by the zero lower bound. (Gertler and Karadi, 2011; Carlstrom et al., 2017; Harrison, 2017; Darracq-Paries and Kuehl, 2017; Sims and Wu, 2019, 2020)[3] By ignoring the occasionally binding nature of financial constraints, previous literature might overstate the usefulness of QE policies in future downturns. Furthermore, the assumption allows us to analyze the optimal conduct of policy after banks' financial constraints stop binding, a relevant question that previous research has neglected.

Second, we analyze the optimal asset-purchase policies under *commitment*. This is different from Gertler and Karadi (2011), who analyze optimal simple asset-purchase rules. We find that the simple rule analyzed in Gertler and Karadi (2011), where the central bank's balance sheet

[3] Bocola (2016) also analyzes quantitative easing policies in a related framework in which banks face occasionally binding balance sheet constraints. Differently from us, however, the paper concentrates on the impact of sovereign risk on the transmission of QE and does not analyze optimal asset-purchase policies.

tracks the interest rate premium (which is a measure of the tightness of *current* balance sheet constraints), converges to the optimal QE policy as the policy coefficient increases towards infinity. We clarify that in the limiting case of such a policy, QE stays active even after the banks' balance sheet constraints stop binding and interest rate premia stabilize at zero. Our exercise also complements the work of Harrison (2017), who solves for optimal discretionary policy. Unlike him, we disregard uncertainty but derive the Ramsey solution to the non-linear problem under perfect foresight. Similarly to him, we find that gradual exit is optimal, but the mechanism is different: in our case, it is driven by the gradual recapitalization of banks, while in his case it is the consequence of households' aversion to quick portfolio-adjustment. Our work is also related to Woodford (2016) and Sims and Wu (2020), who use a stylized model, Ellison and Tischbirek (2014), who analyze optimal combination of simple interest-rate and asset-price rules, and Darracq-Paries and Kuehl (2017), who analyze optimal commitment policies from a timeless perspective. Similarly to these papers, we find that optimal policy uses both interest-rate and quantitative-easing policies also in normal times, and not just in crisis scenarios when the interest rate lower bound is binding.

We structure our paper as follows. In Section 2, we present our model. Section 3 characterizes optimal policy under costless QE, and Section 4 shows the robustness of the results to the presence of a positive cost of QE. We conclude in Section 5.

2 Model

We assess optimal asset purchase and interest rate policies in a New Keynesian model (Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2007) with financial intermediaries (banks, fort short, Gertler and Karadi, 2011, 2013) that face occasionally binding balance sheet constraints. In our exercises, the central bank implements these policies to mitigate the negative impact of a financial disturbance.

The model has seven agents: households, banks, intermediate-good producers, capital-good producers, retail firms, a fiscal authority, and a central bank. Households consume, work, hold short-term deposits at banks and hold long-term government bonds, the latter subject to adjustment cost (Gertler and Karadi, 2013). Banks combine household deposits with equity capital and purchase long-term corporate bonds and long-term government bonds. They face an occasionally binding balance sheet constraint, and they can issue equity subject to adjustment cost (Akinci and Queralto, 2017). Intermediate-good producers issue corporate bonds to finance their capital holdings and use capital and labor to produce intermediate goods. Capital good producers create new capital subject to an investment adjustment cost (Christiano, Eichenbaum and Evans, 2005). Retail firms differentiate intermediate goods and set prices in a staggered fashion a la Calvo (1983). The fiscal authority maintains a fixed supply of long-term government bonds, and the central bank conducts interest rate policy subject to an interest rate lower bound and asset purchase policy subject to a non-negativity constraint.

In this section, we describe the behavior of the banks. Furthermore, we detail selected features of the households' and firms' problems, which make them participate in the corporate- and government bond markets. These features determine the effectiveness of asset-purchase policies. The behavior of the households, the intermediate-good, and capital-good producers, and the retail firms are standard. We collect in Table 1 the equations guiding their behavior and refer the reader to the online appendix for their derivations from the agents' underlying problems.

2.1 Households

There is a continuum of households, and each is comprised of a fraction $1 - f$ of workers and a fraction f of bankers. Workers supply labor, each banker manages a financial intermediary and both return their earnings to the household. The households save by holding long-term government bonds and by depositing funds to intermediaries they do not own. Within the family, there is perfect consumption insurance.

The banker has a finite expected lifetime. Each banker stays a banker with probability σ or becomes a worker with probability $1-\sigma$. The exiting bankers transfer their net worth to their families. We introduce a finite horizon for bankers to ensure that over time they do not save themselves out of their financing constraints. The exiting bankers are replaced by the same number of workers randomly becoming bankers, keeping the relative proportion of each type fixed. The households entrust new bankers with start-up funds, which sum to a potentially time-varying amount ω_t.

The households consume (C_t), supply labor (L_t), and save using two asset types. First, they use deposits and short-term government bonds. They are both one-period real bonds paying a predetermined gross interest R_t from period $t-1$ to t. They are both riskless and therefore perfect substitutes. We denote the households' holding on short-term assets as D_t. Second, the households hold long-term government bonds (B_{ht}) subject to adjustment costs, which we detail below. The standard first-order conditions derived from a standard utility function with internal consumption habits are listed in Table 1.

The long-term government bonds are perpetuities with geometrically decaying coupons: they pay a real coupon of $\varrho^i \Xi$ in periods $i = 0, 1, 2, ...$ Let q_t be the price of the bond. The gross real rate of return on the bond R_{bt+1} is given by

$$R_{bt+1} = \frac{\Xi + \varrho q_{t+1}}{q_t}. \tag{1}$$

We assume that households can hold long-term government bonds subject to a quadratic holding cost that is equal to $\frac{1}{2}\kappa(B_{ht} - \overline{B}_h)^2$ for $B_{ht} \geq \overline{B}_h$. The implied demand for long-term government bond is given by:

$$B_{ht} = \overline{B}_h + \frac{E_t \Lambda_{t,t+1}(R_{bt+1} - R_{t+1})}{\kappa} \tag{2}$$

Demand for long-term bonds above its frictionless capacity level is increasing in the excess return with an elasticity of the inverse of the curvature parameter κ.[4] We allow households to hold long-term government bonds to increase the realism of the model. As we clarify later, the adjustment cost parameter ($\kappa > 0$) directly influences the effectiveness of asset purchases by affecting the share of assets purchased from unconstrained and non-leveraged households as opposed to constrained and leveraged banks.

2.2 Banks

Banks collect short-term liabilities from households and use them, together with their own equity capital, to purchase long-term corporate and government bonds. They can issue new equity subject to adjustment costs.

Long-term corporate bonds provide funding for non-financial firms to finance capital. They can be thought of as equity of the non-financial firms.[5] Let Z_t be the coupon payment from a security that is financing a unit of capital, Q_t, the market value of the security, and δ the depreciation rate of a unit of capital. Then the gross rate of return on the security, R_{kt+1}, is given by:

$$R_{kt+1} = \frac{Z_{t+1} + (1 - \delta)Q_{t+1}}{Q_t} \tag{3}$$

The general equilibrium determines Z_t and Q_t.

2.2.1 The Bank's Problem

Let n_t be the amount of equity capital - or net worth - that a banker has at the end of period t; d_t the deposits the intermediary obtains from households, s_t the quantity of financial claims on non-financial firms that the intermediary holds and b_t the quantity of long-term government bonds. The intermediary balance sheet is then given by

$$Q_t s_t + q_t b_t = n_t + d_t \tag{4}$$

Net worth is accumulated through two sources. First, from retained earnings, which is equal to the difference between the gross return on assets and the cost of liabilities. Second, from the issuance of new bank equity e_t at the end of the period t. New equity is issued by the bank and is financed by the household that owns the bank. The household is willing to transfer

[4] For simplicity, we exclude households from holding long-term private assets. This does not influence our qualitative results, and has only a marginal impact on our quantitative conclusions for the euro area calibration, where only around 10 percent of outstanding liabilities of non-financial corporations are held directly by non-leveraged institutions according to the sectoral accounts.

[5] The conclusions do not rely on corporate bonds defined as equities. For example, Carlstrom et al. (2012) present a version of the model, where corporate bonds are modeled as perpetuities with geometrically decaying coupons and new investment is financed with a new issuance of these bonds. Financial shocks and financial amplification in that model happens analogously to our model: the value of the bond declines similar to the value of capital (equity) in our model to equalize credit supply with credit demand (see also Coenen et al., 2018).

funds to the bank because it enjoys the gross returns of its investment when the bank exits (which can be considered as a stochastic dividend payment). The issuance is subject to a cost $C(e_t, n_t)$ that depends on the size of the bank, and the costs are covered by the household.

$$n_t = R_{kt}Q_{t-1}s_{t-1} + R_{bt}q_{t-1}b_{t-1} - R_t d_{t-1} + e_{t-1} \tag{5}$$

From (4) and (5), net worth can be expressed as

$$n_t = (R_{kt} - R_t)Q_{t-1}s_{t-1} + (R_{bt} - R_t)q_{t-1}b_{t-1} + R_t n_{t-1} + e_{t-1}. \tag{6}$$

The expression makes clear that retained earnings and, therefore, the growth rate of net worth increases with the ex-post excess returns on corporate $(R_{kt} - R_t)$ and government bonds $(R_{bt} - R_t)$.

The banker's objective is to maximize the discounted stream of payouts back to the household, where the relevant discount rate is the household's intertemporal marginal rate of substitution, $\Lambda_{t,t+i}$. To the extent the intermediary faces financial market frictions, it is optimal for the banker to retain earnings until exiting the industry. Accordingly, the banker's objective is to maximize expected terminal wealth. We denote $V_t(s_t, b_t, n_t)$ the end-of-period (after portfolio decisions) value function and $W_t(n_t)$ the beginning-of-period (before portfolio decision, but after shocks) value function of the banker. The end-of-period value function is as follows:

$$V_t(s_t, b_t, n_t) = \max_{e_t} E_t \Lambda_{t,t+1} \left[(1-\sigma)n_{t+1} + \sigma \left(W_{t+1}(n_{t+1}) - e_t - C(e_t, n_t)\right)\right], \tag{7}$$

where we assumed that equity issuance is conditional on the survival of the bank.[6] We further assume that the cost of equity issuance $C(e_t, n_t) = \zeta/2\xi_t^2 n_t$, is linear in the bank's net worth (n_t), quadratic in the share of new equity issuance relative to outstanding equity $\xi_t = e_t/n_t$ and its level is governed by a parameter ζ. The costs realistically disincentivize the bank from large equity issuance. To motivate a limit on the bank's ability to obtain deposits, banks face the following moral hazard problem: At the beginning of the period the banker can choose to divert funds from the assets it holds and transfer the proceeds to the household of which he or she is a member. The cost to the banker is that the depositors can force the intermediary into bankruptcy and recover the remaining fraction of assets.

We assume that it is easier for the bank to divert funds from its holdings of private bonds than from its holding of government bonds: In particular, it can divert the fraction θ of its private bond portfolio and the fraction $\Gamma\theta$ with $0 \leq \Gamma < 1$, from its government bond portfolio.

[6] If the bank exits, it optimally chooses not to issue any equity: it would be repaid to the household right away, but the issuance costs would be lost. Forcing the bank to pay the costs even if it exits would not change our conclusions, only the calibration of the costs.

Accordingly, the depositors supply funds to the banker to make sure that the following incentive constraint is never violated:

$$V_t(s_t, b_t, n_t) \geq \theta Q_t s_t + \Gamma \theta q_t b_t. \tag{8}$$

The left side is what the banker would lose by diverting a fraction of assets. The right side is the gain from doing so.

The bankers maximization problem is to choose its portfolio s_t, b_t to maximize $V_t(s_t, b_t, n_t)$ subject to equation (8), and to choose its equity issuance e_t to maximize $W_{t+1}(n_{t+1})$ subject to equation (6). As is standard in these models, we can verify the existence of a linear solution of the form $V_t = \mu_{st} Q_t s_t + \mu_{bt} q_t b_t + \nu_t n_t$ and $W_t = \eta_t n_t$.

2.2.2 Solution

Let υ_t be the Lagrange multiplier associated with the incentive constraint (8), $\lambda_t = \upsilon_t/(1+\upsilon_t)$ and Ω_{t+1} a term that augments the banks' discount factor relative to the household's discount factor, as we explain below. Then we can characterize the solution as follows.

The expected excess returns on bank assets satisfy

$$E_t\{\Lambda_{t,t+1}\Omega_{t+1}(R_{kt+1} - R_{t+1})\} = \theta\lambda_t \tag{9}$$

When the incentive constraint is not binding, the discounted excess returns are 0. If $\lambda_t = 0, \forall t$, financial markets were frictionless: Banks would acquire assets to the point where the discounted return on each asset equals the discounted cost of deposits. When the incentive constraint is binding, positive excess returns emerge in equilibrium. The excess returns increase with how tightly the incentive constraint binds, as measured by λ_t. Note that the excess return to capital implies that for a given riskless interest rate, the cost of capital is higher than it would otherwise be. As a consequence, investment and real activity will be lower than they would otherwise be. Indeed, a financial shock in the model will involve a sharp increase in the excess return to capital.

The incentive constraint places an (occasionally binding) constraint on the bank's leverage ratio (ϕ_t), which measures its portfolio relative to its net worth:

$$\phi_t = \frac{Q_t s_t + \Gamma q_t b_t}{n_t} \leq \overline{\phi}_t \tag{10}$$

where $\overline{\phi}_t$ is the maximum leverage ratio:

$$\overline{\phi}_t = \frac{E_t \Lambda_{t,t+1}\Omega_{t+1} R_{t+1}}{\theta - E_t \Lambda_{t,t+1}\Omega_{t+1}(R_{kt+1} - R_{t+1})}. \tag{11}$$

The measure of assets that enters the bank's balance sheet constraint applies a weight Γ to government bonds because the government bonds burden the banks' balance sheet capacity less than private assets. This is a direct consequence of the difference in the assets' absconding rates: households are willing to extend more funds to banks that hold more government bonds because they could abscond with less of these. As the bank expands this adjusted measure of assets by issuing deposits, its incentives to divert funds increases. The maximum leverage ratio $\overline{\phi}_t$ limits the portfolio size to the point where the bank's incentive to cheat is exactly balanced by the cost of losing its franchise value.

The bank's maximum leverage ratio $\overline{\phi}_t$ depends inversely on θ, because an increase in the bank's incentive to divert funds reduces the amount depositors are willing to lend. Conversely, an increase in the discounted excess return on assets $E_t\Lambda_{t,t+1}\Omega_{t+1}(R_{kt+1} - R_{t+1})$ (which equals to the bank's partial marginal value of assets μ_{st}) and the discounted safe rate, $E_t\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}$, increases the franchise value of the bank, V_t, reducing the bank's incentive to divert funds. Depositors thus become willing to lend more, raising $\overline{\phi}_t$.

The banks' discount factor is different from the households' discount factor by a multiplicative term $\Omega_t = (1 - \sigma + \sigma\eta_t)$, where η_t is the shadow value of a unit of net worth at the beginning of the period ($\eta_t = V_t/n_t = \partial V_t/\partial n_t$). The term expresses the modified utility value of an extra unit of future income for banks relative to the households. With probability $(1-\sigma)$ the bank exits so the extra income delivers the same utility as an extra income would for the household. With probability σ, however, the bank survives and the extra income raises its net worth, which is valued at the marginal utility $\partial V_t/\partial n_t$. The banks' discount factor is different from the household discount factor whenever the value of bank net worth exceeds unity as a result of the balance sheet constraint that binds or has the potential to bind in the future.

The equilibrium also requires that the banks are indifferent between investing in corporate- and government bonds. Their arbitrage condition is

$$\Gamma E_t\Lambda_{t,t+1}\Omega_{t+1}(R_{kt+1} - R_{t+1}) = E_t\Lambda_{t,t+1}\Omega_{t+1}(R_{bt+1} - R_{t+1}). \tag{12}$$

The condition accounts for the lower relative absconding rate of government bonds, which allows banks to raise more outside funding (face lower margin requirements) for their government bond holdings.[7]

Banks new equity issuance is governed by

$$\xi_t = \frac{E_t\Lambda_{t,t+1}\left(\Omega_{t+1} - 1\right)}{\zeta}. \tag{13}$$

Equity issuance increases with the expected profitability of the bank, which is high when credit conditions are tight and excess premia are high. Additionally, it is sensitive to the adjustment costs: As the adjustment cost parameter ζ approaches zero, the framework approaches a model with flexible equity issuance.

[7]The equality assumes that the bank holds strictly positive quantities from both assets. This requires that the households' and the central bank's demand for long-term government bonds does not exceed the outstanding amount, which would require banks to hold no government bonds in equilibrium. This outcome does not happen under our parametrization for any of our exercises.

2.2.3 Aggregation

Let S_t be the total quantity of corporate bonds that banks intermediate, B_{bt} the total number of government bonds they hold, and N_t their total net worth. In the equilibrium each bank maintains the same leverage (note that their net worth can differ), so we can simply sum across all individual banks (10) to obtain an expression for the aggregate leverage

$$\phi_t = \frac{Q_t S_t + \Gamma q_t B_{bt}}{N_t} \leq \overline{\phi}_t \tag{14}$$

Equation (14) restricts the aggregate banking system leverage to be less than or equal to the maximum leverage. When the constraint is binding, variation in N_t will induce fluctuations in overall asset demand by intermediaries. At the same time, credit supply is restricted and credit costs are strictly above the risk-free deposit rates $E_t\Omega_{t,t+1}(R_{kt+1} - R_{t+1}) > 0$. If the leverage constraint is loose ($\phi_t < \overline{\phi}_t$) then excess returns $E_t\Omega_{t,t+1}(R_{kt+1} - R_{t+1}) = E_t\Omega_{t,t+1}(R_{bt+1} - R_{t+1}) = 0$ are zero.

The total net worth of the banking sector determines the supply of credit in the economy. Total net worth evolves as the sum of retained earnings discounted by the fraction σ of surviving bankers, the new equity issuance and the transfers that new bankers receive, ω_t. We assume that a fraction $1 - \sigma$ of banks exit and repay their net worth to the households. The payback serves as a dividend payout that is proportional to the banks' equity.

$$N_t = \sigma[(R_{kt} - R_t)\frac{Q_{t-1}S_{t-1}}{N_{t-1}} + (R_{bt} - R_t)\frac{q_{t-1}B_{bt-1}}{N_{t-1}} + R_t + \xi_{t-1}]N_{t-1} + \omega_t \tag{15}$$

The first term on the right-hand side describes the retained earnings of banks. Retained earnings are high under tight credit conditions, when excess returns on corporate $(R_{kt} - R_t)$ and government bonds $(R_{bt} - R_t)$ are high. Similarly, incentives to issue new equity (ξ_{t-1}) are high when tight expected future credit conditions raise the expected returns on bank equity. These terms indicate that tight credit conditions are beneficial for the quick recapitalization of the banking sector. Changes in the transfer of new bankers causes exogenous variation in the banking system's aggregate net worth (N_t): $\omega_t = \omega + e_{\omega,t}$, where ω is the steady state value of the transfers and $e_{\omega,t}$ is an iid innovation. The variation is endogenously amplified by the ex-post return on loans R_{kt} and the ex-post return on bonds R_{bt}. Further, the percentage impact of this return variation on N_t in each case, is increasing in the bank's degree of leverage, reflected by the respective ratios of assets to net worth, $Q_{t-1}S_{t-1}/N_{t-1}$ and $q_{t-1}B_{bt-1}/N_{t-1}$.

2.3 Central Bank Asset Purchases

If banks are balance-sheet constrained, excess returns on assets arise with negative consequences for the cost of capital and real activity. Large-scale asset purchases provide a way for the central bank to reduce excess returns and thus mitigate the consequences of a disruption of private intermediation. In particular, we now allow the central bank to purchase long-term government bonds in quantity B_{gt} for a market price q_t.[8]

The purchase reduces the long-term government bond holding of both banks (B_{bt}) and households (B_{ht}). The relative reduction in their holdings is determined endogenously and influence the effectiveness of the policy. The effectiveness of the policy comes primarily from its ability to ease the financial constraints of banks. When banks' financial constraints are binding, the acquisition of bonds from the banks free up their balance sheet and allow them to extend new lending to non-financial corporations. The easier credit conditions stimulate demand, raise the value of both corporate and government bonds and reduce their excess returns.[9] The increase in the value of assets exerts and amplified impact on the levered banks' net worth and further eases their balance sheet constraint in a positive feedback loop. At the same time, the lower excess returns of government bonds reduce the households' willingness to hold this asset making them appear as willing sellers on the long-term government bond market. This reduces the effectiveness of the policy because asset purchases from them (instead of the levered and constrained banks) do not directly ease the credit crunch. The elasticity of households' demand (through the parameter κ) determines the relative share of bonds that needs to be purchased from households. When the demand is elastic (i.e. κ is low) a large share needs to be purchased from households, because a small decline in the premium makes households flexibly release a substantial amount of bonds. Therefore, larger central bank asset purchases are required to achieve the same decline in the premium.

The central bank finances the asset purchases with issuing riskless short-term debt, without facing balance-sheet constraints. At the same time, we assume that the central bank pays a quadratic efficiency cost as τ share of the square of the government bonds that it intermediates ($\tau\left(q_{t-1}B_{gt-1}\right)^2$). This is our reduced-form way to account for unmodeled distortions and political-economy costs of maintaining a large central bank balance sheet. Accordingly, for asset purchases to produce welfare gains, the central bank's advantage in obtaining funds cannot be offset by its costs. Its advantage in obtaining funds is greatest when excess returns are large.

When banks' balance sheet constraints are loose $\phi_t < \overline{\phi}_t$, positive central-bank purchases are ineffective on the margin. In this case, the amount of lending to non-financial corporations (S_t^*) is determined by a no-arbitrage condition $E_t\Omega_{t,t+1}(R_{kt+1}-R_{t+1}) = 0$, and stays unchanged if banks sell part of their government bond holdings to the government and reduce their leverage

[8]For simplicity, we disregard government purchases of corporate bonds. In the euro area, only around 20 percent of the Eurosystem asset holdings originates outside of the public sector (including corporate bonds, covered bonds, and asset-backed securities). Taking their purchases explicitly into account, which can be done straightforwardly in our model, would not change our qualitative conclusions.

[9]The mechanism relies on segmented asset markets and limits to arbitrage as most alternative models of quantitative easing (e.g. Vayanos and Vila, 2009). The segmentation between short-term and long-term government bond markets comes from the fact that banks can abscond with a share of long-term bonds, but not with short-term bonds, and households face portfolio costs for holding long-term government bonds. The banks could eliminate the arbitrage between these markets, but they can be limited in doing so by their equity holdings and leverage constraints.

further. The situation, however, is not symmetric: the policy can turn effective when the central bank resells a sizable chunk of long-term bonds it holds. In this case, the policy can have real effects if it raises banks' leverage so much that it makes the leverage constraints binding again.

2.4 Other agents

In this section, we briefly outline the behavior of the intermediate-good, capital-producing, and retail firms, and the fiscal policy. The equations describing the behavior of these agents, as well as the aggregate resource constraint and the market-clearing conditions, are listed in Table 1. These agents' problems are standard and we relegate the details to the online appendix.

Intermediate-good producing firms issue corporate bonds S_t to finance their capital purchases ($K_{t+1} = S_t$) without any financial frictions. They combine capital with labor (L_t) to produce intermediate goods (Y_{mt}) using a constant returns to scale production function under perfect competition and zero profits. The value of corporate bonds, therefore, is equal to the value of capital Q_t. The availability of credit determines the availability of capital in the economy and therefore influences investment demand (I_t) and production. Capital producers create investment goods using as input final goods and subject to investment adjustment costs. Finally, retailers operate in a monopolistically competitive market, where they use intermediate goods to produce a differentiated good that is combined into the final good as a constant-elasticity of substitution aggregate. As in Rotemberg and Woodford (1997), we assume a constant steady-state subsidy on the revenue of the retail firms. The subsidy is set to offset steady-state distortions caused by the retail firms' market power. Retailers are subject to price rigidities as in Calvo (1983) with backward-looking indexation.

Fiscal policy maintains a fixed supply of long-term government bonds ($\overline{B}$), and finances its expenditures ($G_t = gY_t$) with lump-sum taxes (T_t). Notably, the assumption rules out an active debt-maturity policy, which would adjust the share of government debt financed by long- versus short-term government debt. In theory, the government can achieve the same outcome as large-scale asset purchases by reducing the supply of outstanding long-term government debt.[10] In practice, during the Great Recession, the governments increased the maturity of their debt coincidentally with asset-purchase policies, partly offsetting the aggregate impact of the policy (Robin Greenwood and Summers, 2014). The coordination between a single central bank and multiple debt-maturity agencies in the euro area presents a particularly interesting problem, we leave its analysis for future research. And finally, the aggregate resource constraint covers the costs related to equity issuance, investment adjustment, as well as the costs of large-scale asset purchases.

[10] This would require issuing the same amount of short-term government debt. Short-term government bonds, however, are not subject to financial frictions, i.e. banks cannot abscond with them and therefore can fully finance them from deposits. The holdings of these assets and the deposits raised to finance them can be netted out from banks' balance sheets.

Table 1: Additional model equations

	Household
Marginal utility of consumption	$\mu_t = (C_t - hC_{t-1})^{-1} - \beta_t h E_t (C_{t+1} - hC_t)^{-1}$
Stochastic discount factor	$\Lambda_{t,t+1} = \beta_t \mu_{t+1}/\mu_t$
Euler equation	$E_t \{\Lambda_{t,t+1} R_{t+1}\} = 1$
Labor market equation	$p_{mt}(1-\alpha)Y_t/L_t = \chi \mu_t^{-1} L^{\varphi}$
	Financial Intermediaries
Value of banks' net worth	$\nu_t = E_t\{\left[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1} + (\Lambda_{t,t+1}\Omega_{t+1} - 1)^2/(2\zeta)\right]\}$
Value of a unit wealth	$\eta_t = \nu_t/(1-\lambda_t)$
Lagrange multiplier	$\lambda_t = \max\{1 - \nu_t/(\theta\phi_t), 0\}$
	Intermediate-goods producer
MVP of capital	$Z_t = p_{mt}\alpha Y_{mt}/K_t$
Production function	$Y_{mt} = K_t^{\alpha} L_t^{1-\alpha}$
	Capital goods producer
Optimal investment	$Q_t = 1 + \frac{\eta_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 +$ $\eta_i \left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}} - E_t \beta \Lambda_{t,t+1} \eta_i \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2$
	Retail firms
Retail output	$Y_t = Y_{mt} D_t$
Price dispersion[1]	$D_t = \gamma D_{t-1} \Pi_{t-1}^{-\gamma_P \varepsilon} \Pi_t^{\varepsilon} + (1-\gamma)\left(\frac{1-\gamma \Pi_{t-1}^{\gamma_P(1-\varepsilon)} \Pi_t^{\varepsilon-1}}{1-\gamma}\right)^{-\frac{\varepsilon}{1-\varepsilon}}$
Markup	$P_{mt} = \frac{1}{X_t}$
Optimal price	$\Pi_t^* = \frac{1}{\tau_X}\frac{\varepsilon}{\varepsilon-1}\frac{F_t}{H_t}\Pi_t$
$(\Pi_t^* = P_t^*/P_{t-1})$	$F_t = Y_t P_{mt} + E_t\left[\beta\gamma\Lambda_{t,t+1}\frac{\Pi_t^{-(\gamma_P \varepsilon)}}{\Pi_{t+1}^{-\varepsilon}} F_{t+1}\right]$
	$H_t = Y_t + E_t\left[\beta\gamma\Lambda_{t,t+1}\frac{\Pi_t^{\gamma_P(1-\varepsilon)}}{\Pi_{t+1}^{(1-\varepsilon)}} Z_{t+1}\right].$
Inflation development	$\Pi_t^{(1-\varepsilon)} = \gamma \Pi_{t-1}^{\gamma_P(1-\varepsilon)} + (1-\gamma)\Pi_t^{*1-\varepsilon}$
Fisher equation	$1 + i_t = R_{t+1} E_t \frac{P_{t+1}}{P_t}$
	Fiscal policy
Budget balance	$gY_t + \Xi\overline{B} = T_t + q_t(1-\varrho)\overline{B} +$ $(R_{bt} - R_t) q_{t-1} B_{gt-1} - \tau \left(q_{t-1} B_{gt-1}\right)^2$
	Resource Constraint
Aggregate resource constraint	$Y_t = C_t + [1 + f\left(\frac{I_t}{I_{t-1}}\right)] I_t + \frac{\zeta}{2}{\xi_t}^2 N_t + gY_t + \tau \left(q_{t-1} B_{gt-1}\right)^2$
Capital law of motion	$K_{t+1} = I_t + (1-\delta) K_t$

[1] We thank Huayu Jin (Shanghai University of Finance and Economics) for pointing out a series of typos in the previous version of the equation. We have used the correct equation in our analytical and numerical calculations, so none of our results are affects.

2.5 Central Bank Policy

The central bank sets its policies to maximize the present value of household welfare,

$$W_t = U_t + \beta W_{t+1}, \tag{16}$$

where the utility is separable in labor and consumption internal habits

$$U_t = \log\left(C_t - hC_{t-1}\right) - \chi \frac{L_t^{1+\varphi}}{1+\varphi}, \tag{17}$$

subject to the agents' first order conditions, the market-clearing conditions and the aggregate resource constraints, the occasionally binding balance sheet constraints of banks $\left(\phi_t \leq \bar{\phi}_t\right)$ and any policy constraints: the interest rate lower bound ($i \geq 0$) and positive QE bound ($B_{gt} \geq 0$).

We assume first that the interest rate follows a standard Taylor type rule. In Section 3 and 4 we also show that our results hold under optimal interest rate policy.

$$\exp\{i_t\} = \exp\{i_{t-1}\}^{\rho_i} \left[R^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\kappa_\pi} \left(\frac{Y_t}{Y^*}\right)^{\kappa_Y}\right]^{1-\rho_i} \tag{18}$$

We set the inflation target Π^* consistent with zero inflation and therefore zero price dispersion in the long-run. In the absence of an effective lower bound on the nominal interest rate, the latter is optimal in the Calvo model of price adjustment. This is because inefficient price dispersion generates a distorted allocation of resources across firms or sectors as relative prices vary in ways not justified by sectoral or firm-level shocks, leading to suboptimal quantities of different goods being produced and consumed (Galí, 2015).

The values of Y^* and R^* are set equal to steady-state output and nominal interest rate, respectively. The reason Y^* appears in our measure of output gap instead of the more usual time-varying "natural" level of output is that – in the presence of real imperfections – the *welfare-relevant* output gap is the one between actual output and its *efficient* counterpart (Blanchard and Galí, 2007). And in our model, the efficient level of output remains unaffected by the shock to net worth, which is the main focus of our analysis.

3 Optimal Costless QE

In this section, we characterize key features of the optimal response of the asset purchase policy to a financial shock. We assume that the policy is costless, which we relax in the following section. We describe the results in the form of an assumption, proposition, and corollary and relegate proofs to the appendix.

Assumption. *In the non-stochastic steady state, the financial intermediary sector has sufficient net worth N_{ss} to fully satisfy credit demand. In particular, $N_{ss} \geq N^*$, where $N^* = \left(K_{ss} + \Gamma Bb\right)/\bar{\phi}$ is the steady-state value of the minimum aggregate net worth where the banks' maximum leverage constraint becomes binding.*

Under this assumption, the financial intermediaries' balance sheet constraint is not binding in the non-stochastic steady state. An important implication of the assumption is that the non-stochastic steady state is efficient. Credit is abundant, interest rate premium is zero and output achieves its welfare-maximizing level. Absent credit scarcity, quantitative easing is ineffective, therefore optimal QE is zero.[11]

Proposition. *Under costless asset purchases ($\tau \downarrow 0$) and an adverse financial shock $e_{\omega,t}$, the Ramsey-optimal asset-purchase policy under commitment and perfect foresight is achieved with a rule that is a piecewise linear function of the net worth of the banking sector (N_t). In particular, the optimal value of long-term government bonds in the central bank's balance sheet (qB_{gt}) evolves as*

$$qB_{gt} = \left\{ \begin{array}{ll} \frac{\bar{\phi}}{\Gamma}\left(N^{*} - N_t\right) & \textit{if } N_t \leq N^{*} \\ \mathit{0} & \textit{otherwise} \end{array} \right\},$$

where the variables without subscript denote steady state values. The policy achieves first best by completely offsetting the impact of the shock on the non-financial macroeconomy. The optimal interest rate stays constant at its steady state value.

Proof. In the Appendix. □

Under the optimal policy, the central bank commits to purchasing long-term government bonds in an amount that guarantees that the banking sector can continue to satisfy all demand for private credit. To achieve this, the optimal size of the central bank balance sheet evolves inversely with the banks' net worth gap ($N_t - N^{*}$), a measure of the equity scarcity in the banking sector. The policy responds stronger to the net worth gap if the maximum leverage ($\bar{\phi}$) is higher because then the same gap would generate a larger credit-supply shortfall. In contrast, the policy is less responsive to the gap if the market-risk weight of the long-term government bonds relative to that of private credit (Γ) is higher because then government bond purchases can ease more the risk-weighted balance-sheet constraint of the banks. As the financial shock only hits the banking sector, and the optimal policy fully neutralizes its impact on the macroeconomy, the policy achieves first best.

Along the equilibrium path, excess returns are zero ($R_{kt} - R_t = R_{bt} - R_t = 0$), because the optimal QE policy makes sure that all excess demand for private credit is satisfied. The policy is addictive, because the zero premium reduces the speed of bank recapitalization, and requires asset purchase policies to persist for a long time. Under zero QE costs, we can characterize in closed form the process optimal QE follows. To see this, note that under optimal QE the law

[11] Our main results, including the addictiveness of QE and the slow exit, are robust to relaxing the assumption. First, quantitative easing mitigates the spread after a sufficiently large financial shock irrespective of whether the financial constraints bind in the steady state. The lower spread reduces the speed of bank recapitalization, therefore reduces the speed of optimal QE exit. Second, even if financial constraints are binding in the steady state, a sizable spread is inconsistent with our model. The reason is that in this case, the central bank would have incentives to mitigate the banks' constraints by maintaining a positive balance sheet. The optimal balance sheet size would equalize the marginal welfare gains from mitigating banks' balance sheet constraints with the costs of marginally increasing the size of the balance sheet. If the costs of QE are small, as it is reasonable when considering purchases of long-term government bonds, the central bank would maintain a large balance sheet, and, in parallel, would drive the spread to (very) low levels. Under low spread levels, our benchmark with loose financial constraints and zero spread provides a good approximation for the speed of exit.

of motion of the net worth of the banking sector (15) simplifies to a first-order autoregressive process

$$N_t = \sigma R N_{t-1} + \omega_t.$$

The autoregressive term is the product of the rate of return on net worth (R) and the survival rate of banks (σ). The survival rate determines the retained net worth of the banking sector after a proportional dividend payout, which is a fixed share of the outstanding equity capital (with a proportionality factor $1-\sigma$). Applying the analogous steady-state relationship $N = \sigma RN + \omega$, we can derive the law of motion for the distance of the net worth from its steady state along the optimal path: $N_t - N = \sigma R(N_{t-1} - N) + e_{\omega,t}$, where we assumed a one-off financial shock. The law of motion of the banking sector net worth gap implies the corollary that characterizes the path of optimal QE.

Corollary. *The optimal central bank balance sheet along the equilibrium path follows a first order autoregressive process with a drift until it reaches 0*

$$qB_{gt} = \begin{cases} \sigma R\left(qB_{gt-1}\right) - (1-\sigma R)\frac{\bar{\phi}}{\Gamma}\left(N - N^*\right) - \frac{\bar{\phi}}{\Gamma} e_{\omega,t} & \text{if } N_t \leq N^* \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

with an autoregressive parameter σR and a drift term that depends on the distance between the steady state net worth (N) and its minimum efficient level (N^*).

Proof. In the Appendix. □

Under the optimal policy, banks' balance sheet constraints stay loose, all credit demand is satisfied and interest rate premium stays constant at zero. This is achieved through a QE policy that is large enough to guarantee sufficient credit supply despite the scarcity of banks' equity capital. Exit from QE is very gradual and mirrors the slow recapitalization of the banking sector. This recapitalization is gradual partly because optimal central bank size eliminates excess interest rate premia. This reduces banks' profitability and their incentives to issue new equity, which could speed up the recapitalization of the banking sector, but only at the cost of welfare detrimental aggregate credit scarcity. It should be emphasized that the lack of financial frictions along the equilibrium path does not mean that QE is ineffective. On the contrary, it is the active asset-purchase policy that guarantees that the credit supply satisfies credit demand. Furthermore, in the zero limit of positive QE costs ($\tau \downarrow 0$), the central bank purchases just enough government bonds to make credit supply satisfy credit demand under the maximum steady-state leverage $\bar{\phi}$. Any additional purchase would just increase the efficiency costs without increasing the already fully satisfied credit demand. Therefore, any exit path that is quicker than optimal would necessarily reintroduce credit frictions in the economy and would lead to an economic contraction.

The optimal policy can be implemented in multiple ways. One possibility is to follow the linear rule that responds to the net worth gap of the banking sector as specified in the

Proposition. The rule provides an intuitive target criterion and prescribes a straightforward relationship between the evolution of the central bank balance sheet and the recapitalization of the banking sector. Alternatively, the same equilibrium QE path can be achieved, if the QE policy follows a rule that responds to the excess interest rate premium ($B_{gt} = \nu_R (R_{k,t} - R_t)$) with a response coefficient that increases without limit $\nu_R \uparrow \infty$ (see Figure 2 later). Intuitively, such a policy, if it converges to a finite equilibrium QE path also needs to fully stabilize the premium. In line with the Proposition, full premium stabilization requires the same optimal central bank balance sheet path as described in the Proposition. The relationship between the finite optimal central bank balance sheet path and the zero equilibrium path of the interest rate premium is not as transparent as with the net worth gap rule, however.

4 Optimal Costly QE

In this section, we analyze the robustness of the results described in Section 3, when the central bank asset purchases are costly. We first describe the calibration of the model and the solution method, before presenting our results.

4.1 Calibration and Solution

We calibrate the bulk of the parameters based on Coenen et al. (2018). The parameters are partly calibrated and partly estimated on euro area data using standard Bayesian methods. Table 2 lists the parameter values. Most of the parameters are standard. The discount rate β is calibrated to be 0.995, which implies a steady-state real interest rate of around 2 percent. As in Coenen et al. (2018), the consumption habit parameter is around 0.6, and the Frisch elasticity of labor supply is around 0.5. The capital share α is 0.36, and the capital depreciation rate δ is 0.025. The capital adjustment cost parameter is $\eta_i = 5.17$. The Calvo parameter of price rigidity is $\gamma = 0.92$, with a backward indexation parameter $\gamma_{P,-1} = 0.23$.[12] The duration of the long-term government bond is 7 years ($\varrho = 0.97$), and its overall supply is 70 percent of GDP. We assume that households hold three-quarters of these bonds in the steady state, and banks hold the remaining one quarter.

The calibration of the financial sector follows Coenen et al. (2018) with two important differences. First, we assume that banks' balance sheet constraints are loose in the non-stochastic steady state. In our calibration, banks maintain a buffer in the steady state: their equity capital is around 0.5 percent higher than necessary to fully satisfy credit demand, and their leverage, correspondingly, strictly lower than the maximum leverage. We motivate this choice loosely with precautionary behavior. As shown by Bocola (2016) in a model with a banking sector almost identical to ours, under uncertainty banks optimally choose leverage

[12] The last three parameters are obtained from a previous version of the model (Christoffel et al., 2008), which estimated the model (i) without variable capital utilization, and (ii) without assuming kinked demand curves, which are both missing from our model.

below the non-stochastic maximum. An alternative interpretation of our calibration can come from macroprudential policy, which optimally restricts leverage in our model under uncertainty (Gertler, Kiyotaki and Queralto, 2012). This implies that in the steady state interest rate premium is zero. In contrast, Coenen et al. (2018) assumes that the constraints are binding in the steady state, which leads to a strictly positive interest rate premium. Our choice simplifies the analysis and guarantees that the optimal central bank balance sheet size is zero in the steady state: a positive central bank balance sheet would cause distortions without easing the credit conditions. Our choice, however, does not influence the qualitative conclusions of the analysis of optimal asset-purchase policy conditional on a financial disturbance. As in Coenen et al. (2018), banks' expected planning horizon is 10 years ($\sigma = 0.97$), and we calibrate the fraction of capital that can be diverted θ and the transfer to entering bankers ω such that the interest rate premium is 0, and banks' leverage is just equal to the maximum leverage ($\overline{\phi}$) of 6. The relative absconding rate of government debt relative to private assets (Γ) is 0.83. The household portfolio adjustment cost is $\kappa = 0.009\%$, implying a moderate level of financial frictions. We calibrate the parameter of the cost of equity issuance (ζ) to 28 as in Akinci and Queralto (2017). We show the robustness of our results to this parameter choice.

We calibrate the parameter of the quadratic cost term of QE τ to be 0.1 basis points. The low cost is a reasonable assumption for government bond purchases. Under low costs, large QE interventions stay beneficial in our calibration. This is partly the consequence of our choice to assume an efficient steady state.[13] Our conclusion about the slow optimal exit from large central bank balance sheets stays robust for a realistic range of parameter values.

We analyze optimal interest-rate and asset-purchase policies under commitment and perfect foresight conditional on a bank net worth shock. We solve for the Ramsey-optimal asset-purchase policy subject to three occasionally binding constraints: the banks' balance sheet constraint, the interest rate lower bound, and the minimum asset purchase at zero. We solve the model using the non-linear equation solver for Ramsey-problems in the `dynare` package (Adjemian et al., 2011). We use the package's perfect foresight solver for problems with occasionally binding constraints. The perfect foresight solver implements a Two-point Boundary Value Approach, also known as "extended path" (Fair and Taylor, 1983). The approach relies on two assumptions: (1) certainty equivalence; and (2) the economy reaches steady-state in a finite time. To address inequality constraints, such as the ZLB on the nominal interest rate, the method implements a Levenberg-Marquardt mixed complementarity problem solver following Kanzow and Petra (2004). This is different from Guerrieri and Iacoviello (2015) who propose a piecewise-linear method to tackle inequality constraints such as the ZLB.

[13]This is different from Gertler and Karadi (2011), for example, who consider a distorted steady state and find that large QE interventions are still welfare improving under higher (linear) costs (10 basis points).

Table 2: Parameter values

	Households	
β	0.995	Discount rate
h	0.62	Habit parameter
χ	35	Relative utility weight of labor
B/Y	0.700	Steady state Treasury supply
ϱ	0.97	Geometric decay of government bond
$\bar{B}^h/B$	0.75	Proportion of long term Treasury holdings of the HHs
κ	0.009	Portfolio adjustment cost
φ	2	Inverse Frisch elasticity of labor supply
	Financial Intermediaries	
θ	0.166	Fraction of capital that can be diverted
Γ	0.83	Proportional advantage in absconding rate of government debt
ω	0.067	Transfer to the entering bankers
σ	0.972	Survival rate of the bankers
ζ	28	Parameter of cost of equity issuance
	Intermediate good firms	
α	0.36	Capital share
δ	0.025	Depreciation rate
	Capital Producing Firms	
η_i	5.17	Inverse elasticity of investment to the price of capital
	Retail Firms	
ϵ	3.86	Elasticity of substitution
γ_P	0.92	Probability of keeping the price constant
$\gamma_{P,-1}$	0.23	Price indexation parameter
	Government	
$\frac{G}{Y}$	0.200	Steady state proportion of government expenditures
τ	0.1 basis point	Parameter of the quadratic cost of QE

4.2 Results

The aim of this section is to illustrate the main mechanisms in our model and to show the robustness of our analytical results under a realistic calibration.

Our baseline scenario is a financial recession, in which banks' balance sheet constraints become binding as a result of a negative transitory shock to banks' equity. The size of the drop in banks' equity is of the order of magnitude experienced in the US and in the euro area at the onset of the Great Recession.[14] We first consider a scenario, when asset purchase policies are inactive and interest rate policy follows the standard Taylor rule. The thick black solid lines in Figure 3 plots the impulse responses. The drop in bank equity is sizable enough to make the otherwise loose balance sheet constraint of the banking sector binding. Banks tighten

[14]In the US, for instance, bank equity to total financial assets declined by almost 5 percentage points (from a peak of 8.4% in 2006Q3 to a trough of 3.5% in 2008Q4). In the euro area, the ratio of bank equity to total financial assets fell by almost 4 percentage points (from 18.6% to 14.9% over the same period). Data sources: for USA, FRED II (FBCELLQ027S, and FBTFASQ027S). For Euro Area, Eurostat, consolidated accounts of financial corporations, financial balance sheets (nasa-10-f-bs).

credit, which reduces output by over 0.5 percent and inflation by a bit less than 0.1 percent. The downturn reduces asset valuations, which further depreciates the value of banks' equity in a negative feedback loop. The scarcity of credit raises lending spreads, which increases the profitability of banks. This induces them to issue new equity, which leads to gradual balance sheet repair. In our baseline scenario without policy intervention, the banks' balance sheet constraint stays binding for around 3.5 years.

4.2.1 Zero QE costs

Asset purchase policies are effective and can offset the negative impact of a financial shock. The policy purchases government bonds from households and banks and thereby it relaxes banks' balance sheet constraints. The banks can use their extra balance sheet capacity to offer extra credit to the private sector. The thin black line depicts the optimal asset-purchase policy under zero QE cost. In line with the Proposition, the policy that maximizes household welfare in response to a financial disturbance is one that completely stabilizes the macroeconomy. Namely, inflation and output remain constant, as do asset prices and lending spreads. Bank leverage increases from its steady state to its maximum attainable level while the asset-purchase policy is active.[15] As explained in the Proposition, the policy achieves this by mirroring banks' net worth gap and therefore freeing up sufficient room from banks' balance sheets so as they can fully satisfy credit demand provided they extend their leverage to its maximum. The policy, however, changes the evolution of the banks' net worth. On the one hand, its initial drop is mitigated relative to the no-QE baseline, because asset purchases completely offset the negative feedback of the shock on asset prices. On the other hand, the policy becomes addictive, because asset purchases eliminate lending spreads, thus it reduces the profitability of banks. As a result, new equity issuance will be low, so the recovery in net worth becomes slower than in the absence of asset purchases. The optimal policy, therefore, slows down the balance sheet repair of the banking sector and it contributes to the lengthening of its own duration.

The dashed lines on Figure 2 illustrate that the same optimal outcome can be implemented with a QE rule, which, instead of responding to the net worth gap with an optimally chosen coefficient, responds to the lending spread (as in Gertler and Karadi, 2011) with a coefficient that is increasing without limit. In the limit, the two types of policy rules deliver the same outcome with a persistent QE policy and loose bank balance sheets. Note, however, that unless the coefficient of spread-response is very large, the financial shock produces non-trivial fluctuations in output, inflation, and financial variables, as shown by the red dashed lines.

[15] The maximum attainable leverage is endogenous and depends on the future profitability of the banks. When future profitability, and therefore the franchise value of banks is high, banks are allowed to maintain higher leverage without any risk of absconding with their assets. The optimal policy reduces the expected premium to its steady-state level (zero), therefore stabilizes future profitability and maximum leverage also at their steady-state levels. When the policy is suboptimal, future expected credit scarcity raises banks' expected profitability, therefore allows them to maintain leverage above its steady state. The increase in attainable leverage is insufficient to offset the credit scarcity caused by the drop in bank equity.

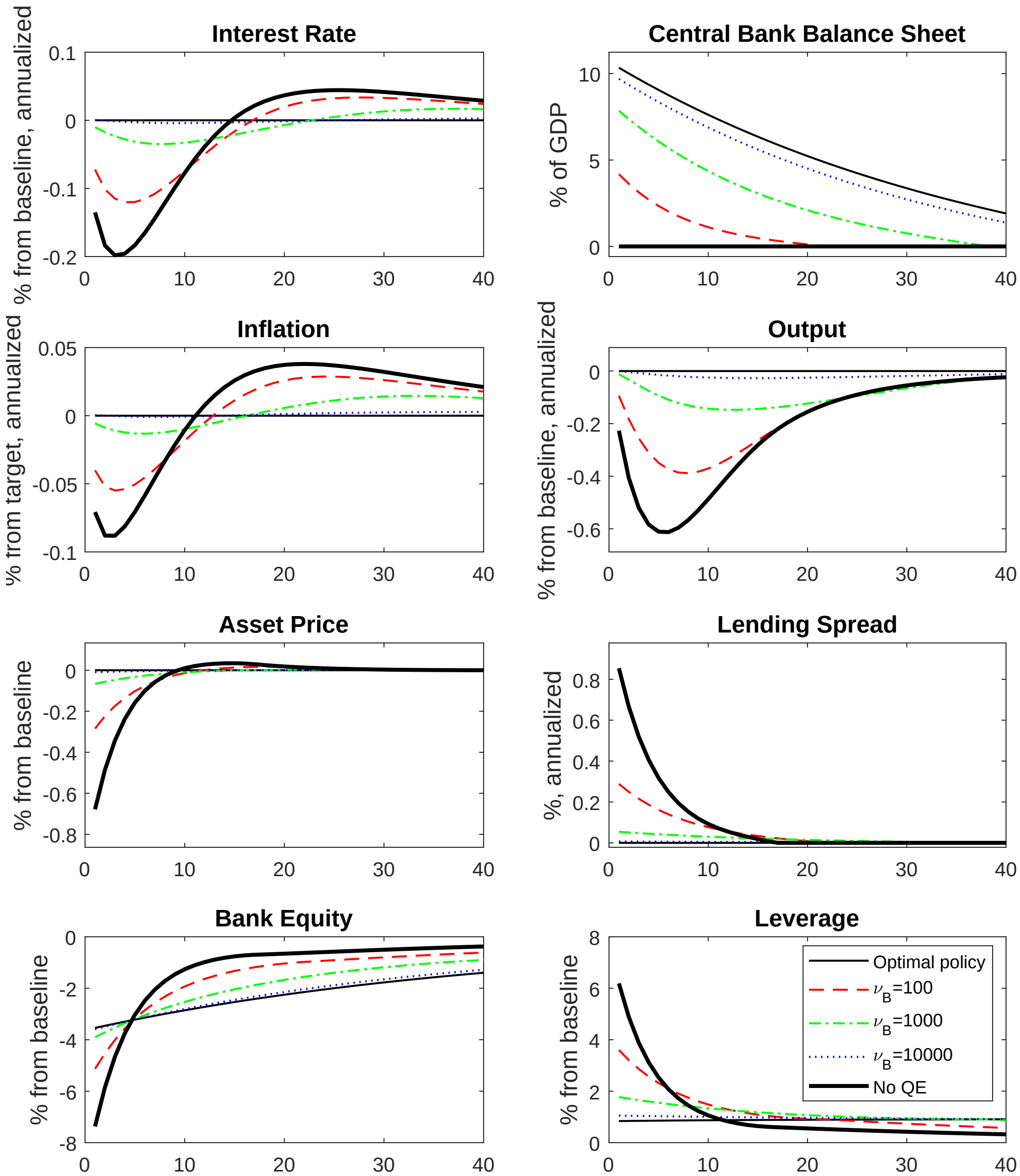

Figure 2: Responses to a financial downturn under no asset purchases (thick solid black line), optimal policy (thin solid black line) and simple rules (dashed lines)

Note: In response to a negative shock to banks' equity, the optimal policy (thin black solid line) raises the central bank balance sheet on impact and commits to undo it very gradually as banks recapitalize. Interest rates stay constant. The figure contrasts the optimal policy to equilibrium paths with simple QE rules (dashed lines) that respond to excess premium with increasing vigour.

Interestingly, before the limit is achieved, the higher aggressiveness of the spread rule, while reducing the cumulative lending spread, extends the period in which the financial constraint stays binding.

4.2.2 Positive QE costs

Next, we turn to the characterization of optimal policy in the more realistic case with a positive QE cost. The black solid line in Figure 3 illustrates that the optimal policy with a positive QE cost is lower than the optimal policy with zero costs (red dashed line), but follows similar dynamics. Importantly, the exit from asset purchases remains very gradual. A notable difference, however, is that the phase-in of the QE policy becomes delayed. Delayed QE entry contributes to a positive lending spread, which leads to a faster initial bank recapitalization. This allows the central bank to save on QE-related costs in the future at a cost of worse financial conditions in the short term. Output, inflation, and interest rates are all nearly constant, while financial variables exhibit some short-lived volatility, except bank equity which is persistently below trend again.

Figure 4 demonstrates the robustness of the high persistence of optimal QE policy if we allow both the interest rate and the asset purchase policy to respond optimally to a financial shock.[16] The black solid lines plot the responses with optimal costly QE, while the red dashed lines correspond to the case without asset purchase policies. The dramatic initial interest rate drop results in inflation and output temporarily above their respective trends, while, as before, bank equity falls below its steady state and the lending spread rises. The asset-purchase policy allows an earlier liftoff from the effective lower bound and helps the economy to achieve more muted fluctuations around the trend. As before, the initial phase-in of QE is delayed and the exit is very gradual.

4.2.3 Welfare effects of QE under different shock and intervention sizes

How should the central bank respond to financial shocks of various sizes? As the Proposition has shown, if QE were costless, the optimal response to financial shocks is to use asset purchase policies without any interest rate easing even if interest rate policy is available. The reason is that these policies address the root cause of the downturn: the credit crunch (see also Carlstrom et al., 2017; Sims and Wu, 2019). With positive QE costs, however, the results are more subtle, and it is far from obvious that embarking on QE is welfare improving even if the downturn is caused by a financial disturbance. We can draw two conclusions based on our model.

First, the central bank should activate the program only if financial shocks are sufficiently large. For small shocks, the banks' balance sheet constraints stay slack, and large scale asset

[16]The size of the financial shock is (three times) larger than in our baseline case with the interest rate Taylor rule. It is calibrated to result in a similar equilibrium drop in bank equity under the more vigorous interest rate easing.

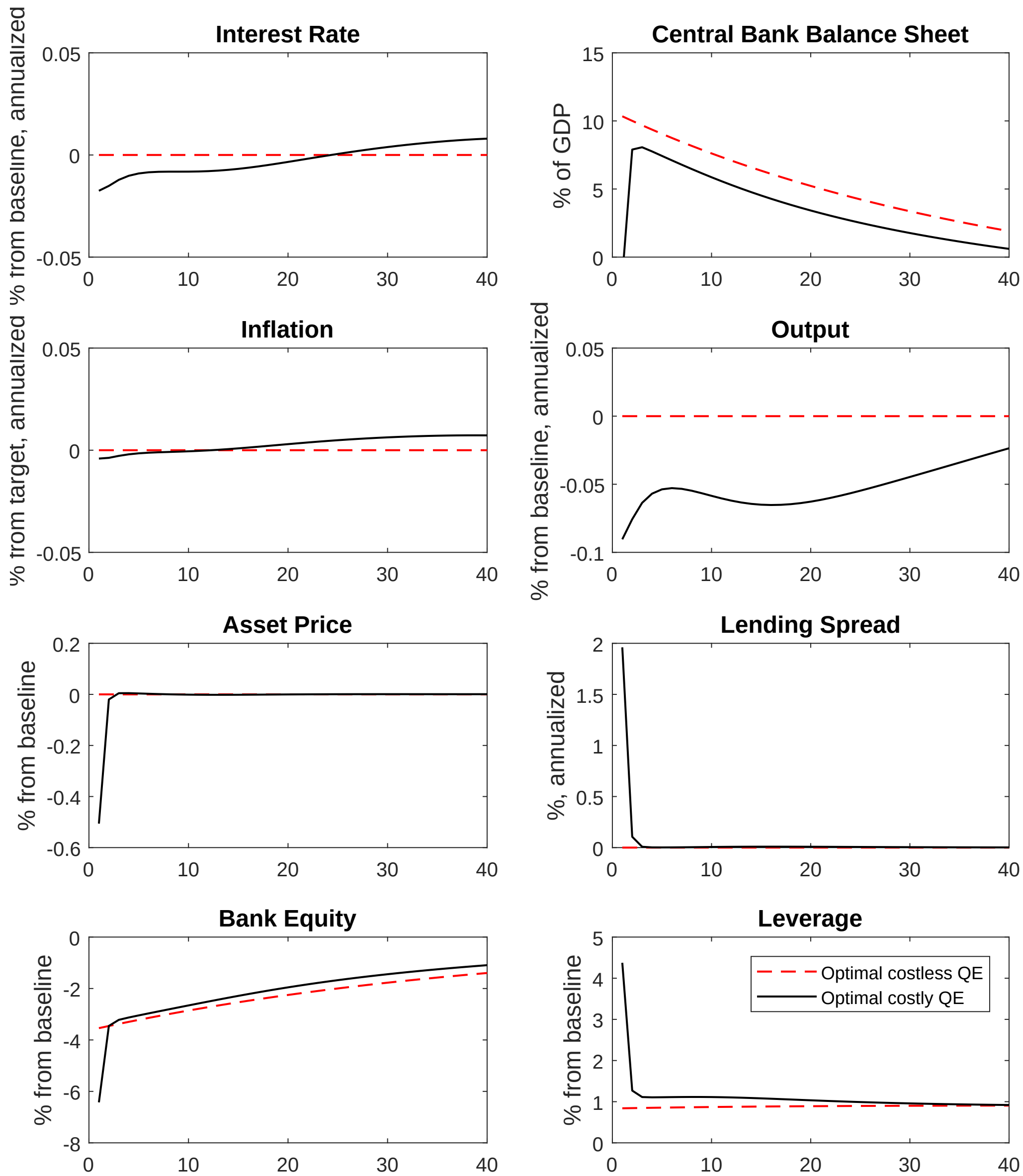

Figure 3: Responses to a financial downturn under zero QE costs (red dashed line) and positive QE costs (black solid line)

Note: The optimal exit from QE is very gradual both under zero and positive costs. While optimal policy insulates the economy from banks' balance sheet problems, it endogenously slows down banks' balance sheet repair through lowering banks' profitability. In the presence of positive costs, the phase-in of the QE becomes delayed.

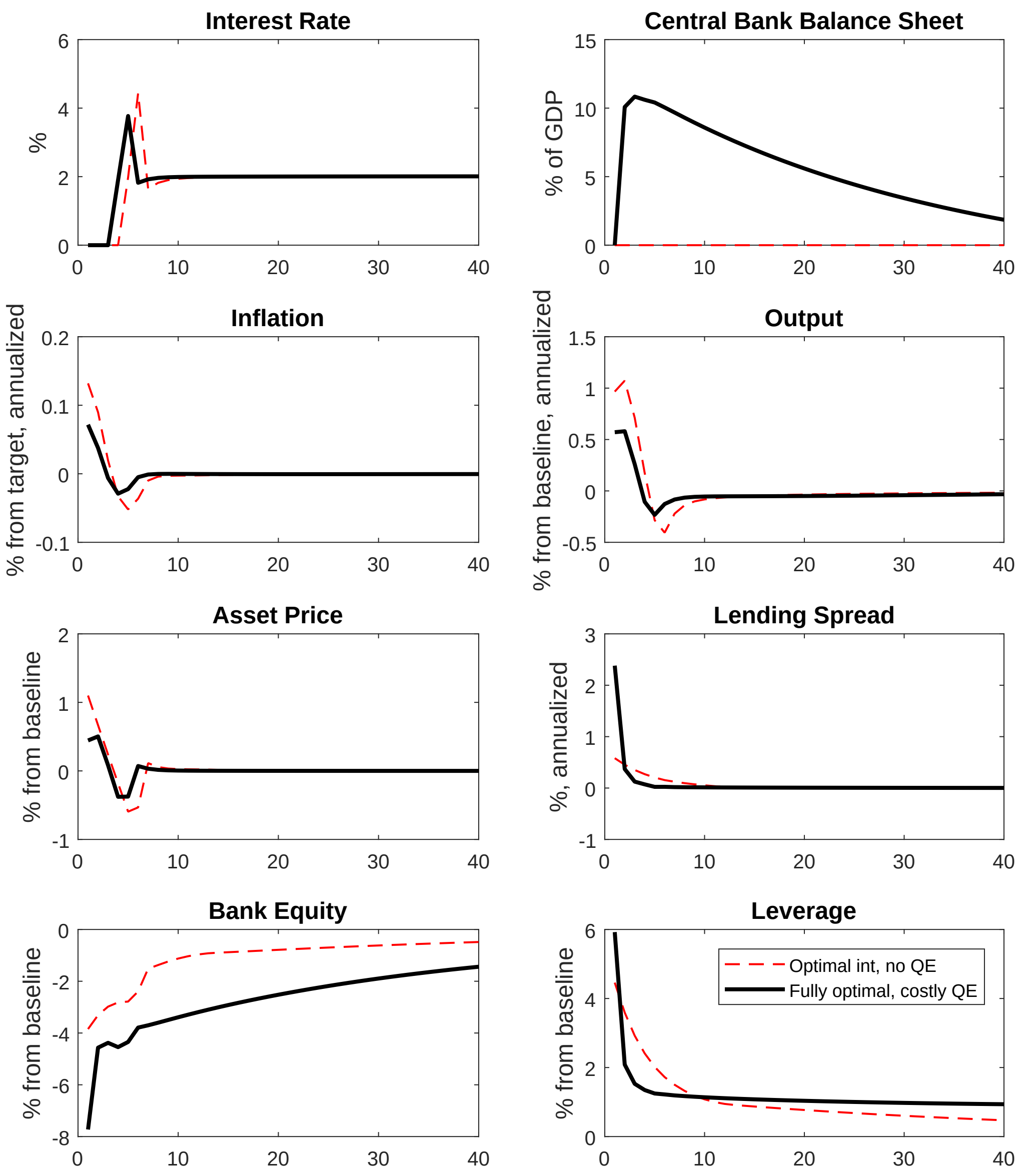

Figure 4: Responses to a financial recession under the optimal combination of interest rate and asset-purchase policy

Note: The optimal exit from QE (solid black line) stays gradual even if interest rate also responds optimally to the financial shock. The active asset purchase policy mitigates fluctuations around the trend and allows an earlier liftoff from the interest rate lower bound.

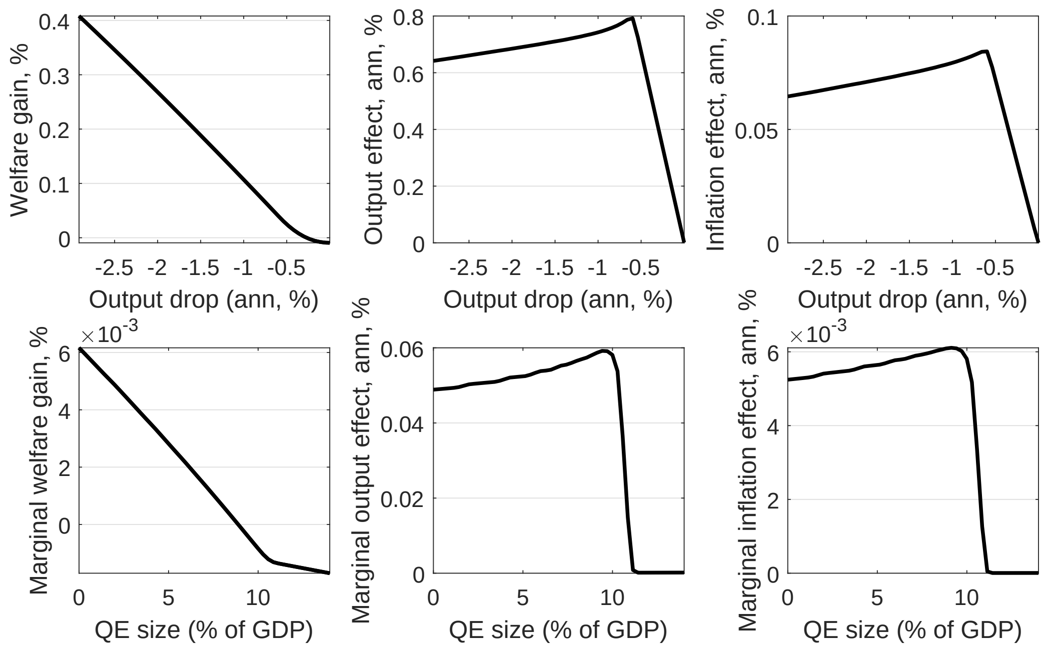

Figure 5: Welfare gains and effectiveness of QE as a function of financial shock size and QE size

Note: The first row of the figure shows the welfare gains, and the output and inflation effects of a QE as a function of different size of financial shocks. The policy follows the dynamics of optimal costless asset purchase policy presented by the Corollary with an initial size of 10 percent of GDP. The size of the financial shock is expressed in terms of the peak output drop. The welfare gains from an infinitesimal shock are negative, and the gains monotonically increase with shock size. The effectiveness of the policy shock increases steeply from zero as the shock size increases, it peaks when financial shock reaches a size where the QE policy cannot completely offset it anymore and falls back somewhat afterward. The second row shows the marginal impact of an additional QE purchase in the amount of 1 percent of GDP under a baseline financial shock as a function of different QE interventions. The welfare gains from marginal QE decrease with QE size and stabilize at a negative value when the QE size completely offsets the financial shock. The effectiveness of a marginal QE initially increases with QE size, but the effectiveness abruptly disappears, after the QE has already fully offset the impact of the financial shock.

purchases have no positive impact on credit creation. Even worse, the efficiency costs of the intervention reduce welfare. As the shock size increases, however, the welfare gains from a fixed-size QE policy intervention increase and become positive, as illustrated by the first panel in the first row of Figure 5. Two factors drive the result. First, as emphasized also in standard quadratic-linear welfare analyzes, the value of a marginal intervention increases as the shock becomes larger and pushes the representative household further away from its optimum. This raises the household's marginal utility, which determines the valuation of the additional intervention. Second, and this is revealed only by our non-linear analysis and illustrated by the second and third panels in the first row of Figure 5: the effectiveness of a large QE intervention can increase fast initially as the shock size increases because it can offset a larger counterfactual downturn. We find that for larger shocks, which are not fully offset by a given size QE intervention, the effectiveness of the intervention decreases somewhat. The reason for this is that without an offsetting QE intervention, the financial shock limits the outstanding credit in the economy. This reduces the strength of the general equilibrium feedback mechanisms triggered by the QE policy intervention because the improvement in outlook and asset valuations brought about by the intervention improves banks' balance sheet conditions to a lesser degree if their outstanding credit is lower. The decline in effectiveness, however, does not offset the increase in the value of the intervention for a realistic range of financial shocks in our parametrization.

Second, the size of the optimal intervention should balance the marginal gains from QE, which declines with the size of the intervention, with its costs. The welfare gains are plotted in the first panel in the second row of Figure 5. The main driving force of the decline, as before, is the declining value of the marginal intervention as the large QE offsets a higher share of the financial shock and brings the households closer to their optimum. As the figure shows, at a certain size of intervention the marginal gains become negative. In these cases, the marginal gains disappear both because financial constraints become slack and the marginal value of intervention declines as households approach their optimum. The declining gains cannot compensate any more the marginal costs of the intervention. The second and third panel in the second row of Figure 5 show that the marginal effectiveness of QE actually increases initially as the QE intervention becomes larger. This happens because, as before, larger aggregate credit supply raises the financial amplification of a marginal intervention. Additional QE becomes ineffective abruptly, however, when the QE intervention fully offsets the financial shock. As before, the initial increase in the effectiveness of the marginal policy intervention is not sufficiently strong to counteract the decline in the value of the intervention.

4.2.4 Non-financial shocks

Up to now, we have restricted our attention to shocks of a financial origin. There are numerous other shocks, however, which can also cause downturns and might even bring the interest rate close to its lower bound. Is it necessarily the case that asset purchase policies are available tools to stabilize the economy when the interest rate becomes constrained? Our model can help to shed some light on this question. Our answer is no, however, and we illustrate this through impulse responses to a persistent shock of the discount factor β_t. A temporary increase in the discount factor raises households' propensity to save and temporarily reduces their consumption demand. The shock reduces the natural rate of interest and monetary policy optimally responds to it by lowering the interest rate.

Figure 6 depicts responses to two such shocks of non-financial origin. These scenarios assume that the interest rate follows a standard Taylor-type rule that is censored at the efficient lower bound, and asset purchase policies are inactive. In both cases, the shock is large enough to make the effective lower bound binding. For the smaller shock (black solid line) the constraint becomes binding for only a couple of quarters and as a result, the interest rate path generates a sufficiently strong easing so as to reduce the real interest rate and offset most of the negative impact of the shock. Importantly, the policy easing appreciates asset prices (through reducing the discount rate) and more than offsets the negative impact of the shock on their cash flow. Banks' balance sheet position actually improves. As a result, financial constraints stay loose and asset purchase policies remain ineffective. The scenario, therefore, provides an example to a downturn when the interest rate lower bound becomes binding, but asset purchase policies cannot help to stabilize the economy.

When the savings propensity shock becomes sufficiently strong, however, such that the interest rate lower bound hinders monetary policy to cut the real interest rate sufficiently strongly, asset purchase policies can again become an effective substitute. In this case, the insufficient monetary easing leads to increasing real interest rates and a sizable downturn, which reduces asset prices. This has an amplified impact on banks' net worth position and leads to binding balance sheet constraints. As in the previous analysis concerning financial shocks, asset purchase policies can be deployed together with the interest rate policy to ease credit conditions and offset the financial amplification channel of the shock.

5 Conclusion

We analyzed the optimal asset-purchase policy in a model with banks, which face occasionally binding financial constraints. We found that the policy is effective but addictive and therefore optimal exit from balance sheet policies is very gradual. Under zero QE costs, the central bank optimally maintains a large balance sheet long after banks' balance sheet constraints stop binding. Furthermore, QE constitutes the optimal response to financial disturbances even if the interest rate is unconstrained by the effective lower bound. Positive costs of the quantitative easing policy, however, justify QE interventions only in response to sufficiently large shocks.

Our analysis warns that QE is not necessarily an effective additional tool when the interest rate is constrained by its lower bound. The reason is that the effectiveness of QE depends on

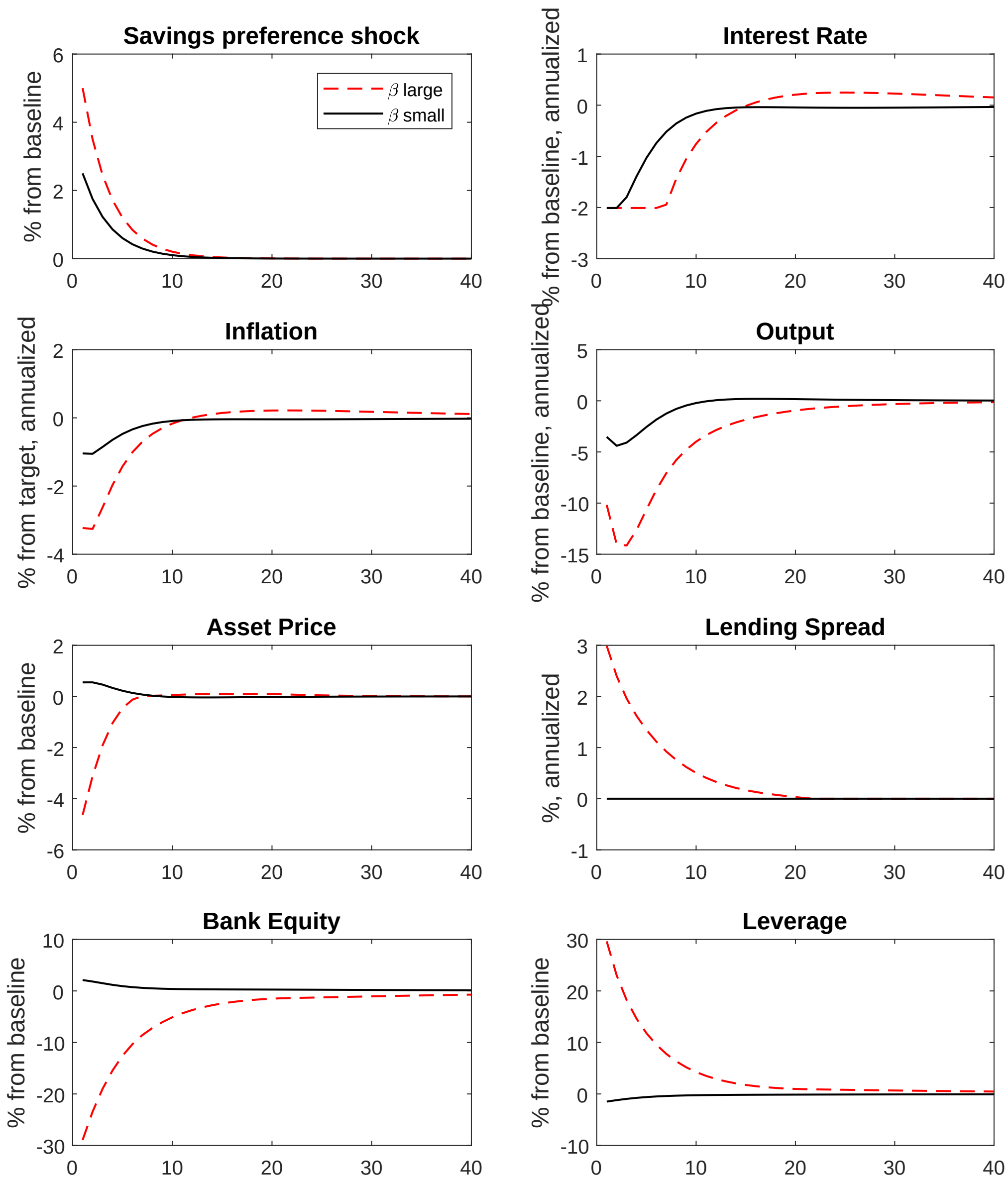

Figure 6: Responses to a non-financial recession

Note: Non-financial shocks can cause downturns where the interest rate lower bound is binding, but asset-purchase policies stay ineffective (red dashed line). The reason is that the policy easing appreciates asset prices despite the downturn. As a result, banks' balance sheet constraints stay loose. When the shock is large enough (solid black line) such that the lower bound restricts interest rate easing sufficiently, the asset-price depreciation caused by the large downturn makes the balance sheet constraints of banks tight and the asset-purchase policy effective.

credit scarcity, which does not necessarily occur when the policy interest rate is constrained. This observation has implications for the effectiveness of QE to avoid steady-state liquidity traps (Benhabib and Schmitt-Grohe, 2001). These are outcomes when interest rate policy is permanently constrained by its lower bound, and inflation stabilizes below target, but other variables, including the credit market, can be at their normal levels. Then, loose balance sheet constraints of the banks can render QE ineffective in driving the economy back towards the preferred steady state.

Our analysis disregarded the role of macro-prudential policies. In our model, subsidies on banks' equity issuance could speed up banks' recapitalization and might permit quicker optimal exit from QE. The optimal policy mix balances the social costs of equity issuance with the costs of QE. The analysis of the optimal interaction of asset-purchase and macroprudential policies, therefore, is an interesting avenue for future research.

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A Proofs

We repeat our main proposition here, before sketching its proof.

Proposition. *Under costless ($\tau \downarrow 0$) asset purchase policy and an adverse financial shock ($e_{\omega,t}$)*[17]*, the optimal asset-purchase policy is achieved with a rule that is a piecewise linear function of the net worth of the banking sector (N_t). In particular, the optimal value of long-term government bonds in the central bank's balance sheet (qB_{gt}) evolves as*

$$qB_{gt} = \left\{ \begin{array}{ll} -\frac{\bar{\phi}}{\Gamma}\left(N_t - N^*\right) & \text{if } N_t \leq N^* \\ 0 & \text{otherwise} \end{array} \right\},$$

where the variables without subscript denote steady state values, and $N^* = \left(K_{ss} + \Gamma Bb\right)/\bar{\phi}$ is the steady state value of the minimum aggregate net worth where the banks' maximum leverage constraint becomes binding. The policy achieves first best by completely offsetting the impact of the shock on the non-financial macroeconomy. Optimal interest rate stays constant at its steady state value.

Proof. The financial shock $e_{\omega,t}$ reduces the net worth of the banking sector. A QE policy, which completely offsets the impact of the shock, and keeps the non-financial sector isolated from the financial turbulence and stable at its nonstochastic steady state achieves the first best. The reason is that the steady state is efficient: all credit demand is satisfied at the slack balance sheet constraint of the financial intermediaries, and any distortions coming from the market power of the retail firms are offset by steady-state subsidies. We guess that such a QE policy is feasible.

Under such policy, the values of the corporate and the government bonds (Q, q), as well as the banking sector corporate bond demand (S_p) stay constant at the steady state. We postulate that the risk-adjusted leverage of the banking system is stabilized at its maximum steady-state level $\bar{\phi}$ while QE is active ($B_{gt} \leq 0$). The constant leverage implicitly defines the necessary evolution of the QE policy

$$\bar{\phi} = \frac{QS_p + \Gamma qB_{pt}}{N_t} = \frac{QS_p + \Gamma q\left(B - B_h - B_{gt}\right)}{N_t},$$

where the second equality has taken into account that the households' demand for government bonds is also constant at the steady state.

The equation implies that $\bar{\phi}N_t = QS_p + \Gamma(qB - B_{gt}) - \Gamma qB_h$. By the definition of N^*, in the steady state $\bar{\phi}N^* = QS_p + \Gamma qB - \Gamma qB_h$. These two equations generate the optimal rule in the proposition. Provided the policy is unconstrained, and the shock is not too large such that the steady-state supply of corporate credit S_p can be maintained, the policy keeps the private lending of the banking system at its steady-state level. The leverage stays constant

[17] Implicitly, we also assume that the shock is small enough such that QE can fully offset it.

at its maximum and the economy stays stable at its efficient steady state while QE is active. After QE policy becomes inactive ($N_t > N^*$), banks further recapitalize and their leverage decreases towards to their steady-state level, while still fully satisfying all credit demand. Any interest rate policy is unnecessary. □

Corollary. *The optimal central bank balance sheet along the equilibrium path follows a first order autoregressive process with a drift until it reaches 0*

$$qB_{gt} = \begin{cases} \sigma R\left(qB_{gt-1}\right) - (1-\sigma R)\,\frac{\bar{\phi}}{\Gamma}\left(N - N^*\right) - \frac{\bar{\phi}}{\Gamma} e_{\omega,t} & \text{if } N_t \leq N^* \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

with an autoregressive parameter σR and a drift term that depends on the distance of the steady state net worth (N) and its minimum efficient level (N^*).

Proof. Under optimal policy, credit premia $R_{kt} - R_t = R_{bt} - R_t$ are stabilized at 0. This implies that the banking system's net worth evolves as $N_t = \sigma R N_{t-1} + \omega_t$. Using the equation for steady state net worth N, we get that

$$N_t - N = \sigma R(N_{t-1} - N) + e_{\omega,t},$$

where $e_{\omega,t} = \omega_t - \omega$. The optimal policy, while active, follows $qB_{gt} = -\bar{\phi}(N_t - N^*)/\Gamma = -\bar{\phi}(N_t - N)/\Gamma - \bar{\phi}(N - N^*)/\Gamma$. As a consequence, $qB_{gt} + \bar{\phi}(N - N^*)/\Gamma = -\bar{\phi}(N_t - N)/\Gamma$. Substituting this equation for both t and $t-1$ into the law of motion of the steady state net worth gap $N_t - N$ implies the corollary. □

B Impact of the QE policy

To illustrate how QE policy works, in Figure 7 we first plot the impulse responses to an exogenous hump-shaped shock to asset purchases. This policy shock hits in our baseline scenario of a financial recession, Without policy intervention, the shock would reduce output by around 0.5 percent and inflation by around 8 basis points. The policy shock is calibrated such that the purchases reach 12 percent of GDP by the 8th quarter, before slowly returning towards zero. The solid black lines show the baseline case when the effective lower bound on the nominal interest rate is not binding, while the red dashed lines are for the case of an interest rate floor that is binding for 4 quarters. In both cases, asset purchase policies are effective in a deep financial recession because banks' balance sheet constraints are binding and the policy frees up banks' balance sheet capacity. This leads to lower lending spreads and more credit to the private sector. The improved credit conditions raise asset prices and improve banks' equity positions, which further eases banks' credit supply constraints in a positive feedback loop. Overall, output increases by about 0.5 percent and inflation by 8 basis points in the case without the ELB. When the ELB is binding, output increases by 1 percent and inflation rises by 16 basis points. These effects are comparable to those found in Coenen et al. (2018) in a linearized model.

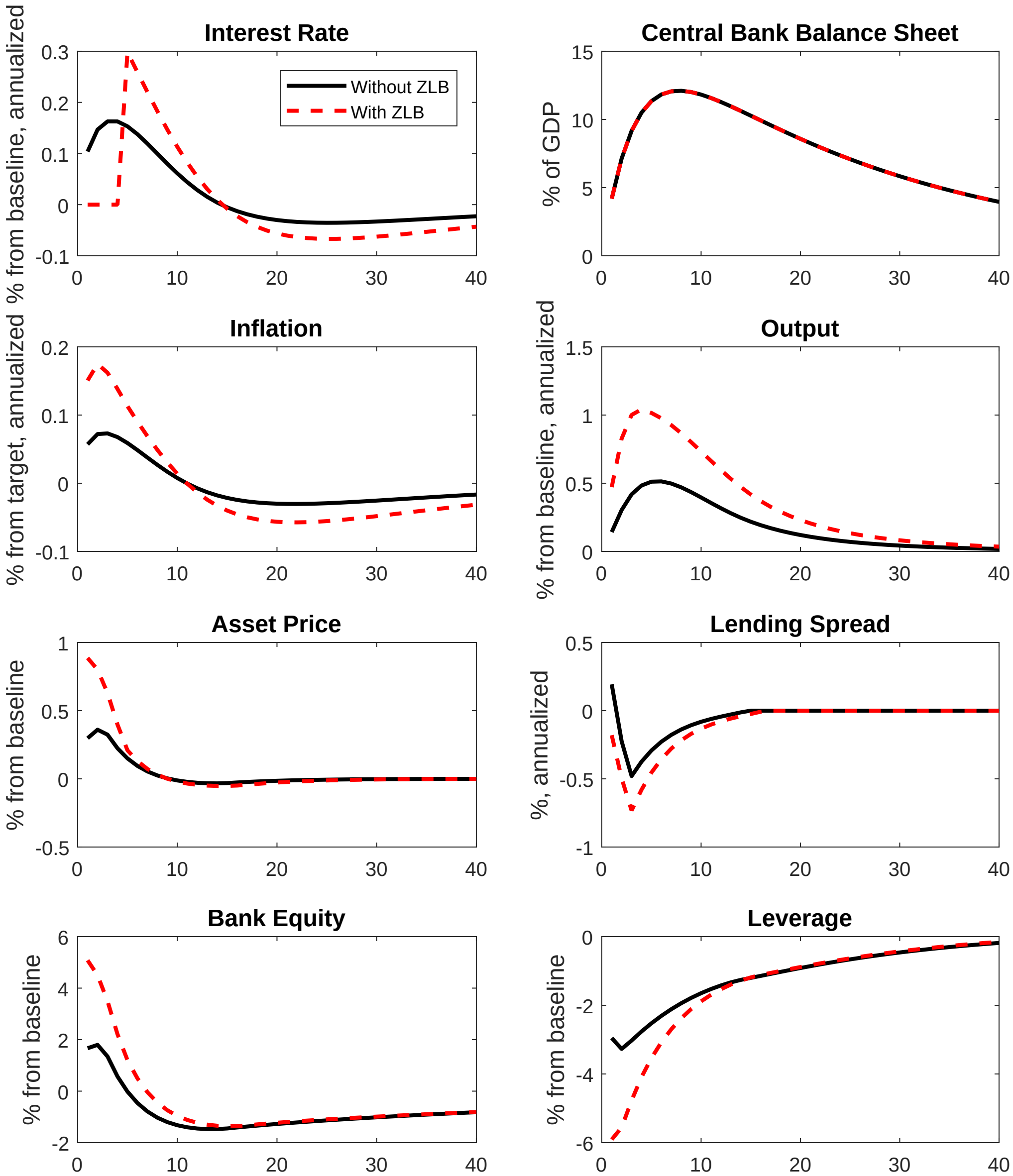

Figure 7: Responses to an asset purchase shock with and without constrained interest rate

C Robustness of the calibration

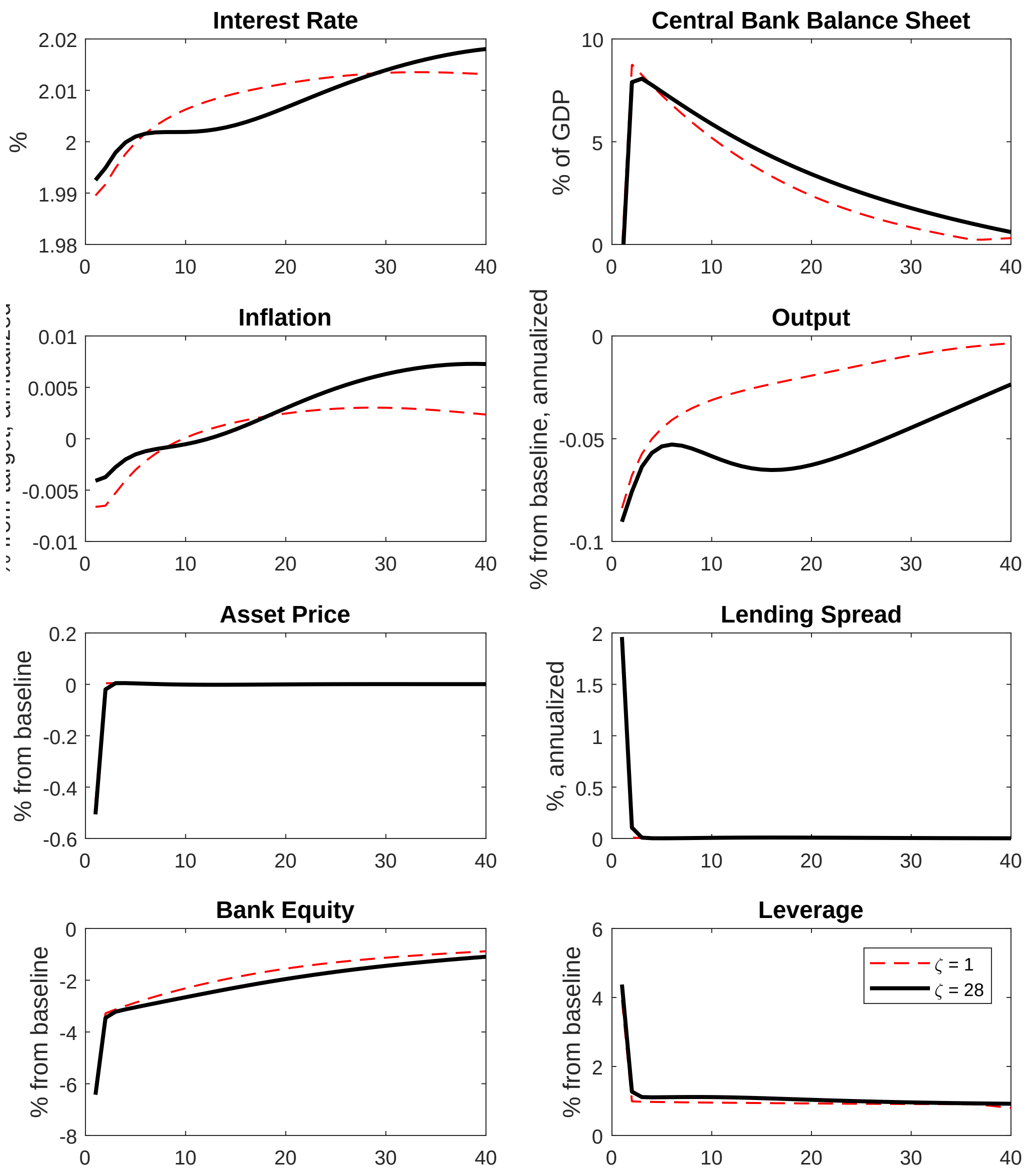

Figure 8: Optimal responses to the baseline financial shock under positive QE costs with varying degree of equity issuance cost parameter

Note: The figure shows that the optimal asset-purchase path is robust to reducing the equity issuance cost parameter from the baseline $\zeta = 28$ level (recovery in 3.5 years without policy) to $\zeta = 1$ (recovery in 1 year without policy).

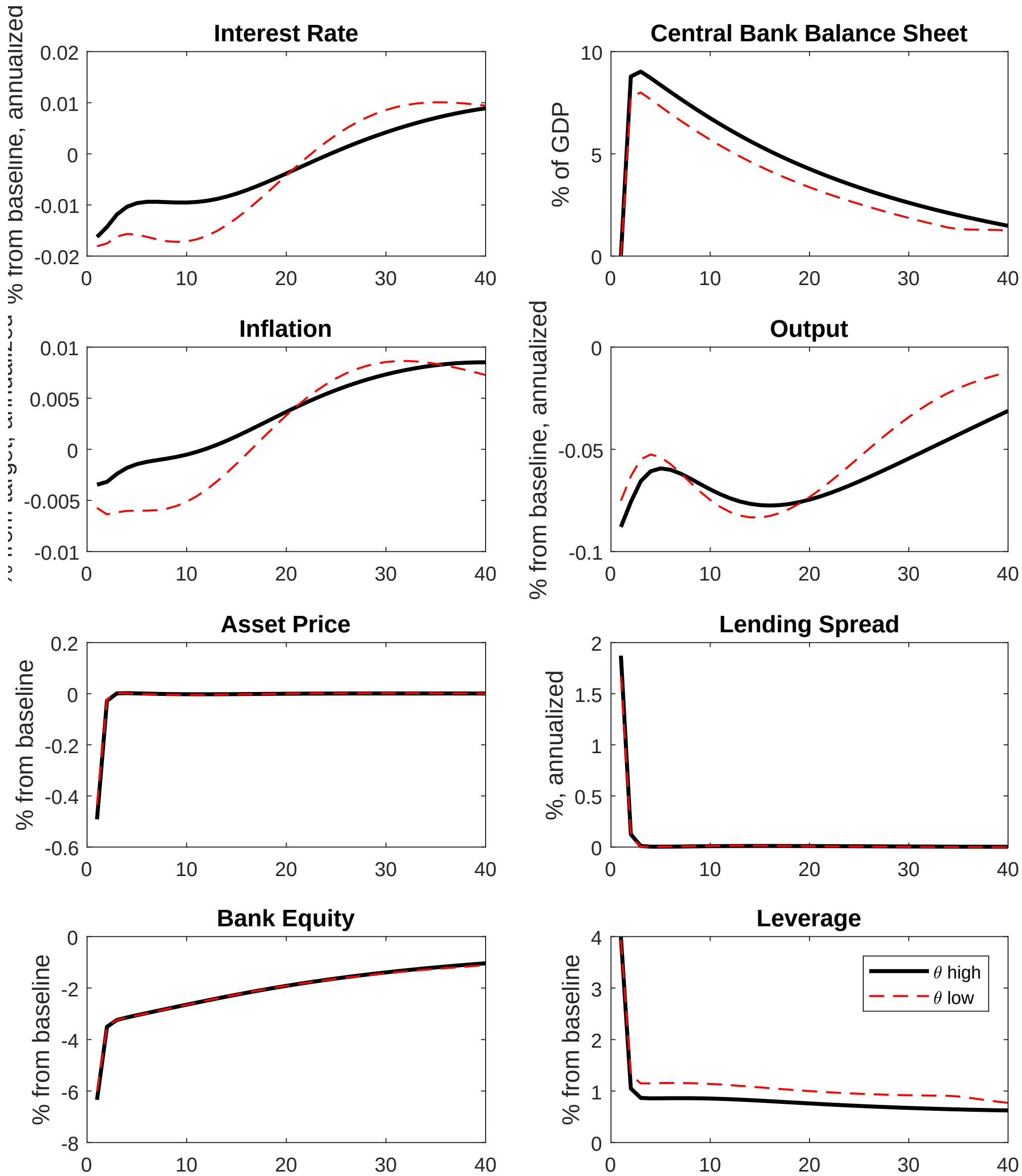

Figure 9: Optimal responses to the baseline financial shock under positive QE costs with varying degree of steady-state equity buffer

Note: The figure shows that the optimal asset-purchase path is robust to increasing the steady state equity buffer from around 0.5 percent to around 1 percent.

D Model

We assess optimal asset purchase and interest rate policies in a New Keynesian model (see e.g. Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2007) with financial intermediaries (banks, for short, Gertler and Karadi (2011, 2013)) that face occasionally binding balance sheet constraints. In our baseline exercises, the central bank implements these policies to mitigate the negative impact of a financial disturbance. The model also takes into account that the interest rate policy can be constrained by a lower bound.

The model has seven agents: households, banks, intermediate-good producers, capital-good producers, retail firms, a fiscal authority, and a central bank. Households consume, work, hold short-term deposits at banks and hold long-term government bonds, the latter subject to adjustment cost (Gertler and Karadi, 2013). Banks combine household deposits with equity capital and purchase long-term corporate bonds and long-term government bonds. They face an occasionally binding balance sheet constraint, and they can issue equity subject to adjustment cost (Akinci and Queralto, 2017). Intermediate-good producers issue corporate bonds to finance their capital holdings and use capital and labor to produce intermediate goods. Capital good producers create new capital subject to an investment adjustment cost (Christiano, Eichenbaum and Evans, 2005). Retail firms differentiate intermediate goods and set prices in a staggered fashion a la Calvo (1983). The fiscal authority maintains a fixed supply of long-term government bonds, and the central bank conducts interest rate policy subject to an interest rate lower bound.

D.1 Households

There is a continuum of identical households of measure unity. Households consume, work, and invest their savings into bank deposits and short-term and long-term government bonds, the latter subject to adjustment costs.

Each household is comprised of a fraction $1 - f$ of workers and a fraction f of bankers. Workers supply labor and return their earnings to the household. Each banker manages a financial intermediary and transfers any earnings similarly back to the household. The banker can choose to issue equity, and the cost of issuance is borne by the household. The households save by holding long-term government bonds (see below) and by depositing funds to intermediaries they do not own. Within the family, there is perfect consumption insurance.

The banker has a finite expected lifetime. Each banker stays a banker with probability σ or becomes a worker with probability $1 - \sigma$, independently of history. The exiting bankers transfer their net worth to their families. We introduce a finite horizon for bankers to ensure that over time they do not save themselves out of their financing constraints. The exiting bankers are replaced by the same number of workers randomly becoming bankers, keeping the relative proportion of each type fixed. The household entrusts new bankers with a small amount of start-up funds. These funds sum to a potentially time-varying amount ω_t across all households. Exogenous time-variation of start-up funds generates fluctuations in the equity position of the banking sector. Financial shocks generated by variation in ω_t will be the focus of our baseline analysis.

Let C_t be consumption and L_t family labor supply. The discounted utility of the household is

$$E_t \sum_{i=0}^{\infty} \beta_t \left[\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \right] \tag{21}$$

with a potentially time-varying discount rate $0 < \beta_t < 1$, $0 < h < 1$ and $\chi, \varphi > 0$.

The household saves by holding deposits, short- and long-term government bonds. Both intermediary deposits and government debt are one period real bonds that pay the gross real return R_t from $t-1$ to t. In the equilibrium we consider, the instruments are both riskless and are thus perfect substitutes. Thus, we impose this equilibrium condition from the outset. We denote the total quantity of short-term debt the household acquires by D_{ht}.

We assume long-term government bonds are perpetuities with geometrically decaying coupons: they pay a real coupon of $\varrho^i \Xi$ in periods $i = 0, 1, 2, ...$ We denote the bond holdings of the household by B_{ht}. Let q_t be the price of the bond. The real rate of return on the bond R_{bt+1} is given by

$$R_{bt+1} = \frac{\Xi + \varrho q_{t+1}}{q_t} \tag{22}$$

The variable q_t are determined in the general equilibrium of the model, as we show later.

While holding short-term assets is costless, we assume that households can hold long-term government bonds subject to transaction costs.[18] In particular, we suppose that for government bonds a household faces a holding cost equal to the percentage $\frac{1}{2}\kappa(B_{ht} - \overline{B}_h)^2/B_{ht}$ of the total value of government bonds held for $B_{ht} \geq \overline{B}_h$.

The household budget constraint is

$$C_t + D_{ht} + q_t[B_{ht} + \frac{1}{2}\kappa(B_{ht} - \overline{B}_h)^2] = \frac{W_t}{P_t} L_t + \Upsilon_t + T_t + R_t D_{ht-1} + R_{bt} q_{t-1} B_{ht-1}, \tag{23}$$

where Υ_t denotes the payouts to the household from ownership of both non-financial and financial firms (including transfer of start-up funds and new equity purchases) and, T_t denotes lump sum taxes.

The household's objective is to choose C_t, D_{ht}, B_{ht} to maximize (21) subject to (23). Let $\mu_t = (C_t - hC_{t-1})^{-1} - \beta_t h(C_{t+1} - hC_t)^{-1}$ denote the marginal utility of consumption. Then the first order conditions for consumption/saving and labor supply are standard:

[18]For simplicity, we exclude households from holding long-term private assets. In the euro area, only around 10 percent of outstanding liabilities of non-financial corporations are held directly by non-leveraged institutions according to the sectoral accounts. Incorporating direct capital holdings into our model would only marginally change our quantitative conclusions.

$$E_t \Lambda_{t,t+1} R_{t+1} = 1 \tag{24}$$

with

$$\Lambda_{t,t+1} \equiv \beta_{t+1} \frac{\mu_{t+1}}{\mu_t}.$$

$$w_t = \chi \mu_t^{-1} L_t^{\varphi} \tag{25}$$

The household's long-term asset demand is given by:

$$B_{ht} = \overline{B}_h + \frac{E_t \Lambda_{t,t+1}(R_{bt+1} - R_{t+1})}{\kappa} \tag{26}$$

Demand for long-term bonds above its frictionless capacity level is increasing in the excess return with an elasticity of the inverse of the curvature parameter κ.

D.2 Banks

Banks collect short-term liabilities from households and use them, together with their own equity capital, to purchase long-term corporate and government bonds. They can issue new equity subject to adjustment costs.

Long-term corporate bonds provide funding for non-financial firms to finance capital. They can be thought of as equity of the non-financial firms. Let Z_t be the coupon payment from a security that is financing a unit of capital, Q_t, the market value of the security, and δ the depreciation rate of a unit of capital. Then the rate of return on the security, R_{kt+1}, is given by:

$$R_{kt+1} = \frac{Z_{t+1} + (1-\delta) Q_{t+1}}{Q_t} \tag{27}$$

The general equilibrium determines Z_t and Q_t.

D.2.1 The Bank's Problem

Let n_t be the amount of equity capital - or net worth - that a banker has at the end of period t; d_t the deposits the intermediary obtains from households, s_t the quantity of financial claims on non-financial firms that the intermediary holds and b_t the quantity of long-term government bonds. The intermediary balance sheet is then given by

$$Q_t s_t + q_t b_t = n_t + d_t \tag{28}$$

Net worth is accumulated through two sources. First, from retained earnings, and second, from issuance of new equity e_t at the end of period t. New equity issuance is subject to a cost $C(e_t, n_t)$ that depends on the size of the bank. It is thus the difference between the gross return on assets and the cost of liabilities:

$$n_t = R_{kt}Q_{t-1}s_{t-1} + R_{bt}q_{t-1}b_{t-1} - R_t d_{t-1} + e_{t-1} \tag{29}$$

From (28) and (29), net worth evolves as

$$n_t = (R_{kt} - R_t)Q_{t-1}s_{t-1} + (R_{bt} - R_t)q_{t-1}b_{t-1} + R_t n_{t-1} + e_{t-1}. \tag{30}$$

The banker's objective is to maximize the discounted stream of payouts back to the household, where the relevant discount rate is the household's intertemporal marginal rate of substitution, $\Lambda_{t,t+i}$. To the extent the intermediary faces financial market frictions, it is optimal for the banker to retain earnings until exiting the industry. Accordingly, the banker's objective is to maximize expected terminal wealth. Furthermore, the bank issues equity e_{t-1}. We denote $V_t(s_t, b_t, n_t)$ the end-of-period (after portfolio decisions) value function and $W_t(n_t)$ the beginning-of-period (after shocks, but before portfolio decision) value function.

$$V_t(s_t, b_t, n_t) = \max_{e_t} E_t \Lambda_{t,t+1} \left[(1-\sigma)n_{t+1} + \sigma \left(W_{t+1}(n_{t+1}) - e_t - C(e_t, n_t)\right)\right] = \tag{31}$$

We assume that the cost of equity issuance $C(e_t, n_t) = \zeta/2\xi_t^2 n_t$, is linear in the bank's net worth (n_t), quadratic in the share of new equity issuance relative to outstanding equity $\xi_t = e_t/n_t$ and its level is governed by a parameter ζ. To motivate a limit on the bank's ability to obtain deposits, banks face the following moral hazard problem: At the beginning of the period the banker can choose to divert funds from the assets it holds and transfer the proceeds to the household of which he or she is a member. The cost to the banker is that the depositors can force the intermediary into bankruptcy and recover the remaining fraction of assets.

We assume that it is easier for the bank to divert funds from its holdings of private loans than from its holding of government bonds: In particular, it can divert the fraction θ of its private loan portfolio and the fraction $\Gamma\theta$ with $0 \leq \Gamma < 1$, from its government bond portfolio.

Accordingly, depositors supply funds to the banker to make sure that the following incentive constraint is never violated:

$$V_t(s_t, b_t, n_t) \geq \theta Q_t s_t + \Gamma\theta q_t b_t. \tag{32}$$

The left side is what the banker would lose by diverting a fraction of assets. The right side is the gain from doing so.

The bankers maximization problem is to choose its portfolio s_t, b_t to maximize $V_t(s_t, b_t, n_t)$ subject to equation (32) with a Lagrange multiplier λ_t, and to choose its equity issuance e_t to maximize $W_{t+1}(n_{t+1})$ subject to equation (30). As is standard in these models, we can verify the existence of a linear solution of the form $V_t = \mu_{st}Q_t s_t + \mu_{bt}q_t b_t + \nu_t n_t$ and $W_t = \eta_t n_t$.

D.2.2 Solution

Let υ_t be the Lagrange multiplier associated with the incentive constraint (8), $\lambda_t = \upsilon_t/(1+\upsilon_t)$ and Ω_{t+1} a term that augments the banks' discount factor relative to the household's discount factor, as we explain below. Then we can characterize the solution as follows.

The expected excess returns on bank assets satisfy

$$E_t\{\Lambda_{t,t+1}\Omega_{t+1}(R_{kt+1} - R_{t+1})\} = \theta\lambda_t \tag{33}$$

When the incentive constraint is not binding, the discounted excess returns are 0. If $\lambda_t = 0, \forall t$, financial markets were frictionless: Banks would acquire assets to the point where the discounted return on each asset equals the discounted cost of deposits. When the incentive constraint is binding, positive excess returns emerge in equilibrium. The excess returns increase with how tightly the incentive constraint binds, as measured by λ_t. Note that the excess return to capital implies that for a given riskless interest rate, the cost of capital is higher than it would otherwise be. As a consequence investment and real activity will be lower than they would otherwise be. Indeed, a financial shock in the model will involve a sharp increase in the excess return to capital.

The incentive constraint places an (occasionally binding) constraint on the bank's leverage ratio (ϕ_t), which measures its portfolio relative to its net worth:

$$\phi_t = \frac{Q_t s_t + \Gamma q_t b_t}{n_t} \leq \overline{\phi}_t \tag{34}$$

where $\overline{\phi}_t$ is the maximum leverage ratio:

$$\overline{\phi}_t = \frac{E_t\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}}{\theta - E_t\Omega_{t,t+1}(R_{kt+1} - R_{t+1})} \tag{35}$$

The measure of assets that enters the bank's balance sheet constraint applies a weight Γ to government bonds because the government bonds burden the banks' balance sheet capacity less than private assets. This is a direct consequence of the difference in the assets' absconding rates: households are willing to extend more funds to banks that hold more government bonds because they could abscond with less of these. As the bank expands this adjusted measure of assets by issuing deposits, its incentives to divert funds increases. The maximum leverage ratio $\overline{\phi}_t$ limits the portfolio size to the point where the bank's incentive to cheat is exactly balanced by the cost of losing its franchise value.

The bank's maximum leverage ratio $\overline{\phi}_t$ depends inversely on θ, because an increase in the bank's incentive to divert funds reduces the amount depositors are willing to lend. Conversely, and increase in the discounted excess return on assets $E_t\Lambda_{t,t+1}\Omega_{t+1}(R_{kt+1} - R_{t+1})$ (which equals to the bank's partial marginal value of assets μ_{st}) and the discounted safe rate,

$E_t\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}$, increases the franchise value of the bank, V_t, reducing the bank's incentive to divert funds. Depositors thus become willing to lend more, raising $\overline{\phi}_t$.

The banks' discount factor is different from the households' discount factor by a multiplicative term $\Omega_t = (1 - \sigma + \sigma\eta_t)$, where η_t is the shadow value of a unit of net worth at the beginning of the period ($\eta_t = V_t/n_t = \partial V_t/\partial n_t$). The term expresses the modified utility value of an extra unit of future income for banks relative to the households. With probability $(1-\sigma)$ the bank exists so the extra income delivers the same utility as an extra income would for the household. With probability σ, however, the bank survives and the extra income raises its net worth, which is valued at the marginal utility $\partial V_t/\partial n_t$. The banks' discount factor is different from the household discount factor whenever the value of bank net worth exceeds unity as a result of the balance sheet constraint that binds or has the potential to bind in the future.

The equilibrium also requires that the banks are indifferent between investing into corporate and into government bonds. Their arbitrage condition is

$$\Gamma E_t\Lambda_{t,t+1}\Omega_{t+1}(R_{kt+1} - R_{t+1}) = E_t\Lambda_{t,t+1}\Omega_{t+1}(R_{bt+1} - R_{t+1}). \tag{36}$$

The condition accounts for the lower relative absconding rate of government bonds, which allows banks to raise more outside funding (face lower margin requirements) for their government bond holdings.

Banks new equity issuance is governed by

$$\xi_t = \frac{E_t\Lambda_{t,t+1}\left(\Omega_{t+1} - 1\right)}{\zeta}. \tag{37}$$

Equity issuance increases with the expected profitability of the bank, and it is sensitive to the adjustment costs. As the adjustment cost parameter ζ approaches zero, the framework approaches a model with flexible equity issuance.

If the leverage constraint is loose ($\phi_t < \overline{\phi}_t$) then excess returns $E_t\Lambda_{t,t+1}\Omega_{t+1}(R_{kt+1} - R_{t+1})$ $= E_t\Lambda_{t,t+1}\Omega_{t+1}(R_{bt+1} - R_{t+1}) = 0$ are zero.

D.2.3 Aggregation

Let S_t be the total quantity of corporate bonds that banks intermediate, B_{bt} the total number of government bonds they hold, and N_t their total net worth. As in the equilibrium each bank maintains the same leverage (note that their net worth can differ), we can simply sum across all individual banks (10) to obtain

$$\phi_t = \frac{Q_tS_t + \Gamma q_t B_{bt}}{N_t} \leq \overline{\phi}_t \tag{38}$$

Equation (38) restricts the aggregate banking system leverage to be less than or equal to the maximum leverage. When the constraint is binding, variation in N_t will induce fluctuations in overall asset demand by intermediaries.

Total net worth evolves as the sum of retained earnings by the fraction σ of surviving bankers, the new equity issuance, and the transfers that new bankers receive, ω_t, as follows.

$$N_t = \sigma[(R_{kt} - R_t)\frac{Q_{t-1}S_{t-1}}{N_{t-1}} + (R_{bt} - R_t)\frac{q_{t-1}B_{bt-1}}{N_{t-1}} + R_t + \xi_{t-1}]N_{t-1} + \omega_t \tag{39}$$

Changes in the transfer of new bankers $\omega_t = \omega + e_{\omega,t}$, where ω is the steady state value of the transfers and $e_{\omega,t}$ is an iid innovation, causes exogenous variation in the banking system's aggregate net worth (N_t). The variation is endogenously amplified by the ex post return on loans R_{kt} and the ex post return on bonds R_{bt}. Further, the percentage impact of this return variation on N_t in each case, is increasing in the bank's degree of leverage, reflected by the respective ratios of assets to net worth, $Q_{t-1}S_{t-1}/N_{t-1}$ and $q_{t-1}B_{bt-1}/N_{t-1}$.

D.3 Central Bank Asset Purchases

If private intermediation is balance-sheet constrained, excess returns on assets arise with negative consequences for the cost of capital and real activity. Within our model, large-scale asset purchases provide a way for the central bank to reduce excess returns and thus mitigate the consequences of a disruption of private intermediation.

In particular, we now allow the central bank to purchase long-term government bonds[19] in quantity B_{gt} for a market price q_t. Let B_t be the total supply of long term government bonds. The purchases will reduce the private holdings of these bonds

$$B_t = B_{bt} + B_{ht} + B_{gt} \tag{40}$$

where as before B_{bt} are the total amounts that are privately intermediated, and B_{ht} is the direct government bond holding of the households determined by equation (26).

When banks' balance sheet constraints are binding the central bank's acquisition of long-term government bonds will bid up the price of this asset. In turn, this will ease banks' constraints, and allow them to extend new lending to non-financial corporations. The easier credit conditions will stimulate demand, raise the value of corporate bonds, and reduce their excess returns. To finance asset purchases, the central bank issues riskless short term debt D_{gt} that pays the safe market interest rate R_{t+1}. In particular, the central bank's balance sheet is given by

$$q_t B_{gt} = D_{gt} \tag{41}$$

[19]For simplicity, we disregard government purchases of corporate bonds. In the euro area, only around 20 percent of the Eurosystem asset holdings originates outside of the public sector (including corporate bonds, covered bonds, and asset-backed securities). Taking their purchases explicitly into account, which can be done with our model, would not change our qualitative conclusions.

where we assume that the central bank turns over any profits to the Treasury and receives transfers to cover any losses. We suppose that the central bank issues the short term debt to households.[20]

These kinds of asset purchases essentially involve substituting central bank intermediation for private intermediation. What gives the central bank an advantage in this situation is that unlike private intermediaries it is able to obtain funds elastically by issuing short-term liabilities. It is able to do so because within our framework the government can always commit credibly to honoring its debt. Accordingly, there is no agency conflict that inhibits the central bank from obtaining funds from the private sector. Put differently, in contrast to private financial intermediation, central bank intermediation is not balance-sheet constrained.

At the same time, we allow the central bank to be less efficient than the private sector at making loans. In particular, we assume the central bank pays a quadratic efficiency cost of τ on the square of the government bonds that it intermediates. Accordingly, for asset purchases to produce welfare gains, the central bank's advantage in obtaining funds cannot be offset by its disadvantage in making loans. Its advantage in obtaining funds is greatest when excess returns are large (i.e when limits to private arbitrage are tight).

When banks' balance sheet constraints are loose $\phi_t < \overline{\phi}_t$, positive purchases ($B_{gt}$) are ineffective on the margin. In this case, the amount of lending to non-financial corporations (S_t^*) is determined by a no-arbitrage condition $E_t\Omega_{t,t+1}(R_{kt+1} - R_{t+1}) = 0$, and stays unchanged if banks sell part of their government bond holdings to the government and reduce their leverage further. The situation, however, is not symmetric: it can turn effective when the central bank resells long-term bonds it holds. In this case, even if banks' balance sheet constraints are loose, the policy can have real effects if it raises banks' leverage so much that it makes the leverage constraints binding again.

D.4 Production, Fiscal Policy, and Equilibrium

We now close the model by describing the non-financial production sector, and the general equilibrium.

There are three types of non-financial firms in the model: intermediate goods producers, capital producers, and monopolistically competitive retailers. The latter is in the model to introduce nominal price rigidities. We describe each in turn.

D.4.1 Intermediate Goods Producers

Intermediate goods producers produce output and sell to retailers. They are competitive and earn zero profits in equilibrium. Each operates a constant returns to scale technology with

[20] Alternatively, we could interpret D_{gt} as interest-bearing reserves (essentially overnight government debt) held by banks on account at the central bank. It is equivalent to our baseline case with short-term debt to households, if we assume that banks cannot abscond with reserves.

capital and labor. Let Y_t be output, A_t total factor productivity, L_t labor, K_t capital, Then:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{42}$$

Let P_{mt} be the relative price of intermediate goods. Then the firm's demand for labor is given by

$$W_t = P_{mt}(1-\alpha)\frac{Y_t}{L_t} \tag{43}$$

It follows that we may express gross profits per unit of capital Z_t as follows:

$$Z_t = P_{mt}\alpha\frac{Y_t}{K_t}. \tag{44}$$

The acquisition of capital works as follows. At the end of any period t, the intermediate goods producer is left with a capital stock of $(1-\delta)K_t$. It then buys I_t units of new capital from capital producers. Its capital stock for $t+1$ is then given by

$$K_{t+1} = I_t + (1-\delta)K_t \tag{45}$$

To finance the new capital, the firm must obtain funding from a bank. For each new unit of capital that it acquires, it issues a state-contingent claim to the future stream of earnings from the unit. Banks are able to perfectly monitor firms and enforce contracts. As a result, through competition, the security that the firm issues is perfectly state-contingent with producers earning zero profits state-by-state. In addition, the value of the security Q_t is equal to the market price of the capital underlying the security. Finally, the period $t+1$ payoff is $Z_{t+1} + (1-\delta)Q_{t+1}$: the sum of gross profits and the value of the leftover capital, which corresponds to the definition of the rate of return in equation (27).

D.4.2 Capital Goods Producers

Capital producers make new capital using as input final goods and subject to adjustment costs. They sell the new capital to firms at the price Q_t. Given that households own capital producers, the objective of a capital producer is to choose I_t to solve:

$$\max E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_\tau^i I_\tau - \left[1 + f\left(\frac{I_\tau}{I_{\tau-1}}\right)\right] I_\tau \right\} \tag{46}$$

From profit maximization, the price of capital goods is equal to the marginal cost of investment goods production as follows,

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - E_t \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right) \tag{47}$$

Profits (which arise only outside of steady state), are redistributed lump sum to households.

D.4.3 Retail Firms

Final output Y_t is a CES composite of a continuum of mass unity of differentiated retail firms, that use the intermediate output as the sole input. The final output composite is given by

$$Y_t = \left[\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{48}$$

where Y_{ft} is output by retailer f.

Retailers simply re-package intermediate output. It takes one unit of intermediate output to make a unit of retail output. The marginal cost is thus the relative intermediate output price P_{mt}. As in Rotemberg and Woodford (1997), we assume a constant steady state tax (ς) on the revenue of the retail firms. The subsidy is set to offset steady state distortions caused by the retail firms' market power ($1-\varsigma = (\varepsilon-1)/\varepsilon$). We introduce nominal rigidities following Calvo. In particular, each period a firm is able to freely adjust its price with probability $1-\gamma$. Accordingly, each firms chooses the reset price P_t^* to maximize expected discounted profits subject to the restriction on the adjustment frequency. Following standard arguments, the first order necessary condition for this problem is given by:

$$E_t \sum_{i=0}^{\infty} \gamma^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} - \mu P_{mt+i} \right] Y_{ft+i} = 0 \tag{49}$$

with $\mu = \frac{1}{1-\varsigma} \frac{\varepsilon}{\varepsilon-1}$. From the law of large numbers, the following relation for the evolution of the price level emerges:

$$P_t = \left[(1-\gamma)(P_t^*)^{1-\varepsilon} + \gamma (P_{t-1})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{50}$$

D.4.4 Fiscal Policy

Government expenditures are composed of: government consumption, which we hold fixed as a share of output at g and the coupon payments from an exogenously fixed stock of long-term government debt, which we set at $\overline{B}$. Revenues consist of lump sum taxes, new issuance of long-term government debt that decays at rate ϱ ($\overline{B} - \varrho\overline{B}$) issued at price q_t and the earnings from asset purchases net of the quadratic transaction costs. Central bank asset purchases are financed by short-term government debt. Given the central bank balance sheet (41), we can express the consolidated government budget constraint as:

$$gY_t + \Xi\overline{B} = T_t + q_t(\overline{B} - \varrho\overline{B}) + (R_{bt} - R_t) q_{t-1} B_{gt-1} - \tau \left(q_{t-1} B_{gt-1} \right)^2 \tag{51}$$

D.4.5 Resource Constraint and Equilibrium

The output is divided between consumption, investment, government consumption, and costs on asset purchases Φ_t. The economy-wide resource constraint is thus given by

$$Y_t = C_t + [1 + f\left(\frac{I_t}{I_{t-1}}\right)]I_t + \frac{\zeta}{2}{\xi_t}^2 N_t + G + \Phi_t \tag{52}$$

with $\Phi_t = \tau\left(q_{t-1}B_{gt-1}\right)^2$.

The link between nominal and real interest rates is given by the Fisher relation

$$1 + i_t = R_{t+1}E_t\frac{P_{t+1}}{P_t} \tag{53}$$

Finally, to close the model, we require market-clearing in markets for private securities, long term government bonds, and labor. The supply of private securities at the end of period t is given by the sum of newly acquired capital I_t and leftover capital $(1-\delta)\,K_t$:

$$S_t = I_t + (1-\delta)\,K_t \tag{54}$$

The supply of long term government bonds is fixed by the government

$$B_t = \overline{B} \tag{55}$$

and labor market clears.

We note that because of Walras' Law, once the markets for goods, labor, and long-term securities clear, the market for riskless short-term debt will be cleared automatically. This completes the description of the model.